Evaluating and reporting astigmatism for individual and aggregate data

Jack T. Holladay, MD, MSEE, Deep R. Dudeja, MD, Douglas D. Koch, MD

ABSTRACT

Purpose: To demonstrate the proper method for evaluating and reporting astigmatism for individual and aggregate data.

Setting: University of Texas Medical School and Cullen Eye Institute, Baylor College of Medicine, Houston, Texas, USA.

Methods: The surgically induced refractive change (SIRC) was determined for three data sets of patients who have had keratorefractive (photorefractive keratectomy) or cataract surgery. To make changes in refraction comparable, vertex distances for the refractions and keratometric index of refraction were considered. Doubled-angle plots and single-angle plots were then used to display the data. Polar values (cylinder and axis) were converted to a Cartesian (x and y) coordinate system to determine the mean value of the induced astigmatism for each data set.

Results: Doubled-angle plots clearly demonstrated the trends of induced astigmatism for each data set, and the mean value for induced astigmatism agreed exactly with the intuitive appearance of the plot.

Conclusions: Converting astigmatism data to a Cartesian coordinate system allowed the correct computation of descriptive statistics such as mean values, standard deviations, and correlation coefficients. Using doubled-angle plots to display the data provides the investigator with the best method of recognizing trends in the data. J Cataract Refract Surg 1998; 24:57–65

In 1992,1 we described the method for calculating surgically induced spherical and astigmatic change, which is required to correctly evaluate existing and evolving corneal surgical techniques. The mathematics

From the University of Texas Medical School, Houston (Holladay, Dudeja), and Cullen Eye Institute, Department of Ophthalmology, Baylor College of Medicine, Houston, Texas (Koch).

Mark W. Riggs, PhD, Department of Biostatistics, Scott & White Memorial Hospital, Temple, Texas, provided the graphics for the doubled-angle, single-angle, and equivalency plots.

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Reprint requests to Jack T. Holladay, MD, Houston Eye Associates Building, 2855 Gramercy, Houston, Texas 77025, USA. for calculating the surgically induced refractive change (SIRC), first described over 100 years ago,² were properly applied some 125 years later by Jaffe and Clayman³ to determine the effects of various types of sutures, incisions, and suturing techniques to explain and refine their surgical techniques for cataract surgery. The various approximation methods that have been described⁴-7 can lead to significant errors in the analysis and lead to erroneous conclusions regarding the results.

Although there are several available computer programs that use the exact mathematical solution to calculate the SIRC, approximation methods that lead to significant errors in the analysis and erroneous conclusions continue to emerge. Furthermore, presentations

at the annual meetings of the American Society of Cataract and Refractive Surgery, European Society of Cataract and Refractive Surgeons, and American Academy of Ophthalmology describe "homemade" methods for evaluating and reporting astigmatism results. Since our original article, there has been no change in the method for calculating the SIRC on an individual case, but improved methods of evaluating, reporting, and displaying aggregate data that make the conclusions derived from the data much more intuitive and useful have been developed. Using actual data sets from refractive and cataract surgery, we will develop the theoretical basis for these new methods and illustrate their usefulness in understanding and reporting astigmatism data.

Materials and Methods

A method for calculating the SIRC for an individual case from keratometric and refractive data was discussed in our original article and will not be repeated. However, whether the calculations are performed graphically using vectors or computationally using Cartesian coordinates or polar coordinates, the resulting SIRC must be the same or there has been a mistake. To use mathematical jargon, there is one and only one SIRC for preoperative and postoperative data from a single patient, regardless of the method used. This is analogous to determining the length of the side of a right-angle triangle given the other two sides; one may use Cartesian coordinates, polar coordinates, trigonometric functions, or the Pythagorean theorem. No matter what method is used, the result must be the same.

Basics of Astigmatism

Every SIRC or spherocylinder can be written in one of three ways: plus cylinder, minus cylinder, or crossed cylinder form. Although each form is identical in value, there are advantages to using one form over another when analyzing data. For example, if we are analyzing arcuate keratotomies (AK) or T-cuts (TK), in which the direct effect of the surgery is corneal flattening, the minus cylinder form is most applicable because the negative sign indicates a reduction in power that corresponds to the relative flattening that has occurred in the surgical meridian. If compressive sutures or

wedge resection were performed, steepening will have occurred in the surgical meridian and the plus cylinder form is more appropriate for illustrating the effect of the surgery. In each of these examples, the cross cylinder form describes the exact effect in each of the orthogonal meridians. This form is helpful in describing the change in each meridian independently to determine whether there has been a coupling effect (steepening in the opposite meridian to the meridian of the flattening procedure and vice versa). For example, the three forms of the SIRC for change in refraction of a patient who had an eight-incision radial keratotomy (RK) are

Plus cyl axis form: $-5.34 + 1.44 \times 65$ Minus cyl axis form: $-3.90 - 1.44 \times 155$ Cross cyl axis form: -3.90×65 and -5.34×155

Surgically induced refractive changes that are a result of changes in the cornea are best visualized when expressed in the power notation (@) rather than axis (\times) form. Expressing refractive changes in the power notation also allows direct comparison with keratometric changes, since keratometric data are always in power notation. The power notation is always 90 degrees to the axis notation; e.g., $+1.00 \times 90 = +1.00$ @ 180. Rewriting the SIRC above in the power form, we have

Plus cyl power form: -5.34 +1.44 @ 155

Minus cyl power form: -3.90 -1.44 @ 65

Cross cyl power form -3.90 @ 155 and -5.34 @ 65

From this SIRC in the power form, we see that by flattening the meridian of 155 degrees by 3.90 diopters (D) and the meridian of 65 degrees by 5.34 D, the surgeon has induced 1.44 D of astigmatism. Although the plus cylinder power form is correct and equivalent to the other two forms, it is misleading because this procedure flattened all meridians.

Vertexing Spherocylinders to the Corneal Plane

Refractions are normally performed at the spectacle plane or in the phoropter and not at the corneal plane. For SIRCs determined by refraction to be compared with SIRCs determined by keratometry or topography, they must be vertexed to the corneal plane. Refractions performed in the phoropter are at a vertex of 13.75 mm when the corneal vertex is located at the large mark on the vertex calibration scale. Spectacles are usually 1.0 or

2.0 mm closer and would therefore be at a vertex distance of approximately 12.0 mm.

To vertex spherocylindric refractions to the corneal plane, the spherocylinder must first be converted to the cross cylinder form. Vertex calculations cannot be performed with either plus or minus spherocylindric form. This is because the cylinder in these forms does not represent the power in either meridian; rather, it is simply the difference in powers and therefore may not be vertexed. In the cross cylinder form, the cylinders represent the actual power in each meridian and therefore may be vertexed.

In the above example, if we assume that the refraction was performed in the phoropter at approximately 14.0 mm, we can calculate the spherocylindric refraction at the corneal plane in the following manner:

Cross cyl power form

@ spectacle: -3.90 @ 155 and -5.34 @ 65

Vertex: 14.0 mm

Vertex formula from spectacle plane (REF_S) to corneal plane (REF_C), 8.9

$$REF_{C} = \frac{1000 * REF_{S}}{1000 - REF_{S} * Vertex (mm)}$$
(1)

Using the above values, we have

$$REF_{CI} = \frac{1000 * (-3.90)}{1000 - (-3.90) * 14} = -3.70 D$$

$$REF_{C2} = \frac{1000 * (-5.34)}{1000 - (-5.34) * 14} = -4.97 \text{ D}$$

Cross cyl power form

@ cornea: -3.70 @ 155 and -4.97 @ 65

Vertex: 0 mm

Plus cyl power form: -4.97 + 1.27 @ 155

Minus cyl power form: -3.70 -1.27 @ 65

From this example, we see that when correctly vertexed to the cornea, the astigmatism at the corneal plane is almost one-quarter diopter less. This relationship from the spectacle plane to the corneal plane for astigmatism is always true for compound myopia; i.e., the astigmatism is always less at the corneal plane than at the spectacle plane for compound myopia. For compound hyperopia, the relationship is just the opposite; i.e., the astigmatism is always more at the corneal plane than at the spectacle plane. This vertex conver-

sion of the SIRC from spectacle plane to the corneal plane must be performed before keratometric or topographic data are comparable.

Special Considerations with Keratometric Data

Although keratometric data are already at the corneal plane and do not require any vertex considerations, a different problem arises with the index of refraction used to convert the anterior radius of cornea to a refractive power. The formula used to convert radii to power is the simple spherical refracting surface formula (SSRS):

$$K_{k} = \frac{n_2 - n_1}{r} \tag{2a}$$

The variables n_1 and n_2 are the indices of refraction of the first and second media, respectively, and r is the radius of curvature of the interface. The value for n_1 is 1.000 (index of refraction for air), and the standardized keratometric index of refraction (1.3375) was chosen for n_2 many years ago. The origin of this value for n_2 remains obscure, dating back to the 19th century. However, it appears to have been arbitrarily selected so an anterior radius of corneal curvature of 7.5 mm would yield a power of 45.0 D.¹⁰ We can think of no other rationale for choosing the index of refraction of 1.3375 other than to make these two numbers (7.5 mm and 45 D) agree exactly.

$$K_k = \frac{1.3375 - 1.000}{r_a} = \frac{0.3375}{r_a}$$
 (2b)

where r_a is the anterior radius of curvature of the cornea and K_k is the standardized keratometric corneal power.

The cornea, like any meniscus lens, has a front surface power, a back surface power, and an equivalent or net power. The average index of refraction of the cornea is 1.376 and the index of refraction of the tears and aqueous, 1.336.¹¹ To properly compute the change in power of the cornea, one must know whether the changes are front surface power changes, back surface power changes, or net power changes. The front surface power and net power changes are the only clinically relevant considerations since there are no keratorefractive procedures intended to change only the back surface power.

For procedures such as photorefractive keratectomy (PRK), laser in situ keratomileusis (LASIK), and prob-

ably RK, the change in dioptric power of the cornea is almost entirely due to front surface power changes in the cornea. To compute front surface power, the change in media for the light rays is from air (n=1.000) to cornea (n=1.376), so, as Holladay and Waring¹⁰ and Mandell¹² have recommended, the correct formula for computing the power and any change in power would be

$$K_a = \frac{1.376 - 1.000}{r_a} = \frac{0.376}{r_a}$$
 (2c)

where r_n is the anterior radius of curvature of the cornea and K_n is the front surface corneal power. The front surface power of a cornea with an anterior radius of 7.5 mm would be 50.13 D (0.376/0.0075), 5.13 D greater than the standardized keratometric power of 45.00 D. Front surface powers are 11.14% (0.376/0.3375) larger than keratometric values. When the change in refractive power is being determined for keratorefractive procedures that change the front surface only, the change in refractive power computed from keratometry using the standardized keratometric index must be increased by 11.14% to be accurate.

For analyzing results in which both front and back surfaces have been changed equally, it is appropriate to use the net or equivalent corneal index of refraction. The most common application of this conversion is for intraocular lens (IOL) power calculations. Unlike agreement on values in computing the front surface power, there is still some debate among investigators as to the most appropriate value for the net or equivalent index of refraction. The value determined empirically from several thousand cases by Binkhorst¹³ was 4/3 (1.3333...); the lowest value, 1.3315, was determined by Olson.¹⁴ For standardization purposes, 15 we have recommended adopting Binkhorst's value since it is the predominant value that has been used for over 20 years. The equation to compute net or equivalent corneal power, using indices of refraction for air (n = 1.000) and cornea (n = 4/3), would be

$$K_n = \frac{4/3 - 1.000}{r_n} = \frac{1/3}{r_n}$$
 (2d)

where r_a is the anterior radius of curvature of the cornea and K_n is the net corneal power. The net corneal power of a cornea with an anterior radius of 7.5 mm would be

44.44 D ($\frac{1}{3} \div 0.0075$), 0.56 D less than the standardized keratometric power of 45.00 D. Net or equivalent corneal powers are 98.76% ($\frac{1}{3} \div 0.3375$) of the standardized keratometric values. When the net or equivalent power of the cornea or changes in the net corneal power are needed, the standardized keratometric values must be reduced to 98.76% of their original values to accurately reflect the net refractive power change in the cornea.

For net powers of the cornea used in IOL calculations, a 1.24% overestimate of the corneal power results in a 0.56 D error, which is significant and intolerable for these calculations. However, for calculating changes in corneal power produced by refractive surgical procedures, this 1.24% error is clinically negligible. Nevertheless, when reporting net corneal power changes from standardized keratometry measurement, the values should be reduced by 1.24% to be correct.

It is important to note that the value of 1.333 for net corneal index of refraction may change following refractive surgical procedures that alter epithelial thickness or remove corneal tissue (i.e., PRK and LASIK). The refractive indices of the epithelium and stroma are different, and there may also be subtle intrastromal differences, e.g., between Bowman's layer and the posterior stroma. Procedures that alter epithelial thickness could change the refractive power of the epithelium, and removal of stromal tissue could alter the net refractive index of the stroma. These changes could be important in determining the correct value for net corneal power for IOL calculations, since, as noted, a change of only 1 to 2% could produce unacceptably high errors.

Calculating Prediction Error from the Desired and Actual Postoperative Refraction

Excimer lasers, toric IOLs, and incisional surgery can induce spherical and astigmatic changes in the refraction. In most cases, the goal of refractive surgery is to neutralize the spherocylindric correction so the final postoperative refraction is plano. When this occurs, the desired SIRC and the actual SIRC must be equal. When the final refraction does not match the desired postoperative refraction, the desired SIRC and the actual SIRC must be different.

The definition for prediction error is given by the equation

Although the difference between the desired postoperative refraction and the actual postoperative refraction must be calculated like any other SIRC, it can usually be done easily because the desired postoperative refraction is spherical, so the solution for obliquely crossed cylinders is not necessary.

To illustrate this concept, let us consider a 50-year-old patient with a refractive error of $-5.00 +3.00 \times 90$ vertexed to the corneal plane. We are planning to perform LASIK and leave the patient -0.50 D myopic to provide some vision in the midrange as well as good distance vision. Our desired SIRC at the corneal plane is therefore $-4.50 +3.00 \times 90$, one-half diopter less than the refractive error at the corneal plane. The patient's actual postoperative refraction at 1 month vertexed to the corneal plane was $-0.50 +0.50 \times 80$. Since the desired postoperative refraction was -0.50 D, substituting equation 3 the prediction error is

Prediction Error = Desired Postop Ref
- Actual Postop Ref
Prediction Error =
$$(-0.50) - (-0.50 + 0.50 \times 80)$$
= plano -0.50×80

The value of the difference between the desired and the actual postoperative refraction is plano -0.50×80 (-0.50 @ 170). This value indicates that the error from the desired target was one-half diopter of cylinder at an axis 80 or @ 170. The prediction error and SIRC can be treated similarly when analyzing aggregate data.

Analyzing Aggregate Data

Once the data of a group of patients have been vertexed to the corneal plane and standardized keratometric data converted to front surface or net corneal power, evaluation and display of aggregate data can begin. In our original description, we suggested that spheroequivalent (SEQ) data can be averaged in the normal manner and displayed on a plot in which the x-axis is a logarithmic time scale, i.e., the distance from 1 to 10 days is the same interval as 10 to 100 days (Figure 1). Descriptive statistics such as means, standard deviations, standard error of the means, and correlation coefficients are calculated in the normal manner. Spheroequivalent statistics and graphs are par-

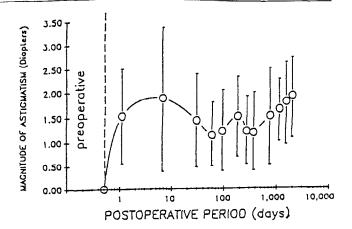


Figure 1. (Holladay) Spherical equivalent for a 41 patient data set following cataract surgery is shown for various postoperative periods (O is the mean and I is the standard deviation). A negative spherical equivalent indicates a decrease or weakening (hyperopic direction) of the corneal power following the cataract procedure; a positive value, an increase or strengthening (myopic direction). The spherical equivalent never changed by more than 0.25 D.

ticularly valuable for analysis of procedures in which no induced astigmatism was intended, such as spherical refractive surgery (spherical PRK or LASIK).

Another method is to display the spheroequivalent of the SIRC. The spheroequivalent of any spherocylinder is the sphere plus one half the cylinder. The formula is

Spheroequivalent = Sphere +
$$\frac{1}{2}$$
 * Cylinder (4)

A plot of spheroequivalent SIRC data is shown in Figure 2A on an equivalency plot. When data points are on the diagonal, the desired and actual refractions are equal. When the result is above the diagonal, there is an overcorrection; when below the diagonal, an undercorrection. The prediction error may be plotted versus the desired SEQ SIRC as shown in Figure 2B. The information displayed is similar to the equivalency plot, but the exact values for the errors are easier to see.

The magnitude of astigmatism (cylindric power in the plus or minus cylinder form) may be analyzed in a similar manner, but the information about the axis of the astigmatism is lost (Figure 3). From the magnitude of astigmatism plots alone, one cannot infer any trends such as against-the-rule or with-the-rule astigmatism changes from the data. The most appropriate method for evaluating, reporting, and displaying astigmatism data requires conversion of the

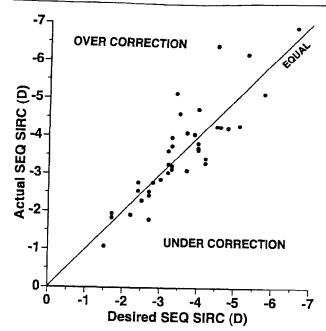


Figure 2A. (Holladay) Equivalency plot showing the actual spheroequivalent (SEQ) of the surgically induced refractive change (SIRC) plotted against the desired SEQ of the SIRC. The diagonal line is where the actual and desired SEQ of the SIRC are equal. The actual data from a 43 patient data set for the period between 3 and 6 months postoperatively are shown. Data points above the diagonal are overcorrections and those below the diagonal, undercorrections.

data from our normal method of describing astigmatism in polar coordinates (cylinder and axis) to a Cartesian coordinate system.

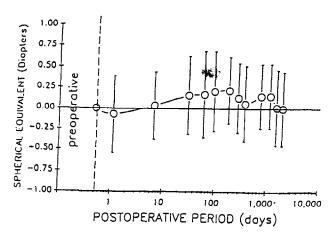


Figure 3. (Holladay) The magnitude of astigmatism for a 41 patient data set following cataract surgery is shown for various postoperative periods (O is the mean and I is the standard deviation). The magnitude of the astigmatism for each patient in a given period is averaged. In this data set, the smallest amount of astigmatism was present between 60 and 365 days.

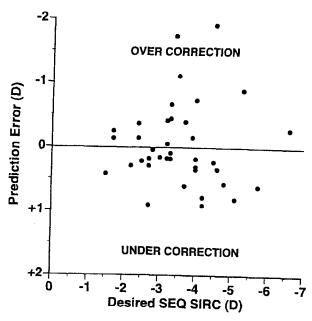


Figure 2B. (Holladay) The prediction error is defined as the difference between the desired postoperative refraction and the actual postoperative refraction. The actual data from a 43 patient data set for the period between 3 and 6 months postoperatively are shown. Data points above the horizontal are overcorrections and those below the horizontal, undercorrections.

Evaluating, Reporting, and Displaying Aggregate Astigmatism Data

Astigmatism data are difficult to analyze primarily because of the way astigmatism is defined. The axis of astigmatism returns to the same value when it traverses an angle of 180 degrees, whereas in geometry and trigonometry, one must traverse an angle of 360 degrees to return to the same point. To apply conventional geometry, trigonometry, and vector analysis to astigmatism, the angles of astigmatism must be doubled so that 0 degree and 180 degrees are equivalent. Once this transformation has been performed, all standard methods of vectors and geometry and trigonometry are applicable and produce the correct singular value for the SIRC.

Because astigmatism traverses an entire cycle in 180 degrees, the most appropriate plot of aggregate astigmatism data is a doubled-angle polar plot (Figure 4). The doubled-angle plot goes from 0 to 180 degrees in a full cycle, rather than from 0 to 360 degrees, with 45 degrees at 12 o'clock, 90 degrees at 9 o'clock, 135 degrees at 6 o'clock, and 180 degrees back at

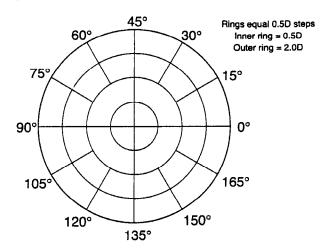


Figure 4. (Holladay) The doubled-angle plot is a polar plot of astigmatism data using the value of the cylinder for the magnitude and the axis of the astigmatism for the angle. The angles range from 0 to 180 degrees and correspond to the range of angles for astigmatism. The rings represent the magnitude of the astigmatism; the inner ring is 0.5 D, the outer ring, 2.0 D, and the step size between rings, 0.5 D.

3 o'clock. The 0 and the 180 degree locations are the same, just as 0 and 180 degrees are the same for the axis of refraction. On a standard polar plot, 0 and 180 degrees are on opposite ends of the x-axis, which does not correspond to the relationship of 0 and 180 degrees for the axis of astigmatism. Any procedure that on the average is astigmatically neutral must have the centroid of the data at the center of the plot. On a single-angle plot (standard polar plot), none of these statements is true.

Determining the Mean Cylinder and Axis for Induced Astigmatism

In our original article, we presented a method for determining the average axis or meridian of astigmatism. We have since recognized that this method is incorrect because it does not appropriately incorporate both the magnitude and axis of the SIRC into the calculations. For standard descriptive statistics (i.e., means, standard deviations) to be applied correctly, each data point must be converted to an x-y coordinate system. Descriptive statistics cannot be applied to polar coordinates because the two components of the polar coordinate form—cylinder magnitude and axis—are not orthogonal parameters. Descriptive statistics require the components to be orthogonal like the x and y

axes. This conversion to different values is similar to visual acuity measurements that must be converted to LogMAR values before descriptive statistics can be applied. To convert a cylinder and axis to Cartesian coordinates, the following formula should be used.

$$x = \text{Cylinder} * \text{Cos}(2 * \text{axis})$$
 (5a)

$$y = \text{Cylinder} * \text{Sin}(2 * \text{axis})$$
 (5b)

In the formulas, we must double the angle of the axis of astigmatism to determine the correct values for x and y.

Results

Figure 5A is a set of 43 cases of spherical PRK on the Summit laser for which the induced astigmatism has been plotted using the calculated x and y values for each point on a doubled-angle minus cylinder power plot. It is easy to see there is a trend toward 0 degree. To illustrate the point, Figure 5B shows the same data on a single-angle plus cylinder plot, where the trend is more difficult to discern.

Once we have converted all the data points to a Cartesian coordinate system, the standard descriptive statistic formulas may be applied. To determine the centroid or mean value of a set of x and y values, the mean value is the mean of the x and y values independently. In equation form

Mean of
$$X = \frac{\sum_{i=1}^{n} x_i}{n}$$
 (6a)

Mean of
$$Y = \frac{\sum_{i=1}^{n} y_i}{n}$$
 (6b)

Applying these formulas to the PRK data set, we find the mean value of X to be 0.399 D and Y to be 0.022 D. To convert from Cartesian coordinates back to the standard polar notation for astigmatism, we have

Cylinder =
$$\sqrt{X^2 + Y^2}$$
 (7a)

Angle =
$$\frac{1}{2}$$
 * Arc $\tan\left(\frac{Y}{X}\right)$ (7b)

IF
$$X \& Y > 0$$
 THEN Axis = Angle (7c)

IF
$$X < 0$$
 THEN Axis = Angle + 90° (7d)

IF
$$X > 0 & Y < 0$$
 THEN Axis = Angle + 180° (7e)

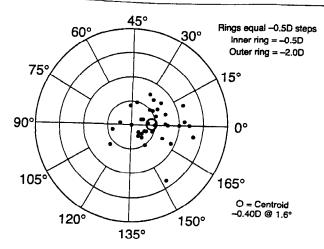


Figure 5A. (Holladay) A doubled-angle, minus cylinder power plot of a 43 patient data set for the postoperative period between 3 and 6 months after PRK is shown. It is clear from the doubled-angle plot that there is a tendency toward more flattening in the horizontal meridian. The exact mean astigmatism (centroid) is -0.40 D @ 1.6.

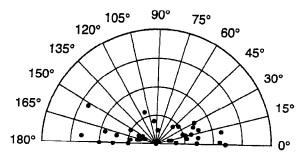


Figure 5B. (Holladay) A single-angle, minus cylinder power plot of the 43 patient data set in Figure 5A shows no trend in the data. The centroid is meaningless and therefore is not shown. This plot illustrates why single-angle cylinder plots that have historically been used are of little value.

Substituting our values for x and y, we have

Cylinder =
$$\sqrt{.399^2 + .022^2}$$
 = 0.40 D

Angle =
$$\frac{1}{2}$$
 * Arc tan $(\frac{0.022}{0.399})$ = 1.6°, since $x \& y > 0$

The mean value of the induced astigmatism is -0.40 @ 1.6. It is clear from these data that the spherical ablation with the Summit excimer laser has induced an average -0.40 D of astigmatism at 1.6 degrees. This point intuitively appears to be at the center of the cluster of the data displayed in the doubled-angle plot. Analysis by any other method will lead to erroneous results.

Figure 6A illustrates induced with-the-rule astigmatism on the first day after single-suture, small incision cataract surgery. Figure 6B illustrates obliquely induced astigmatism at 135 degrees by a right-handed surgeon who always displaces his incision to the right (superotemporal in right eye and superonasal in left eye) to facilitate his use of the phaco handpiece.

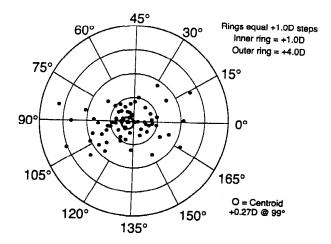


Figure 6A. (Holladay) A doubled-angle, plus cylinder power plot of a 64 patient data set illustrates induced steepening of the vertical meridian 1 month after single-suture, small incision cataract surgery. The preponderance of data is near the 90 degree meridian. The exact mean astigmatism (centroid) is +0.27 @ 99.

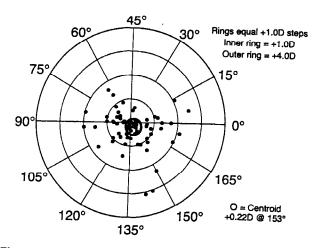


Figure 6B. (Holladay) A doubled-angle plus cylinder power plot of a 63 patient data set illustrates obliquely induced astigmatism at the 153 degree meridian by a right-handed cataract surgeon 1 month after surgery. He always displaces his incision to the right (superotemporal in right eye and superonasal in left eye) to facilitate his use of the phaco handpiece. The exact mean astigmatism (centroid) is +0.22 @ 153.

Discussion

The method for calculating the SIRC for an individual patient is a unique solution that should not be approximated with some of the incorrect computational tools that have been used. Before any analysis, one must correct for the vertex distance for refractions and convert standardized keratometric data to front surface power or net power. Doubled-angle formulas and plots are necessary to compute and display astigmatic data accurately since astigmatism completes a cycle in 180 degrees rather than the normal 360 degrees. Descriptive statistics such as means, standard deviations, standard error of the mean, and correlation coefficients must be performed after the astigmatism data have been converted to Cartesian coordinates (x and y values). After computations have been performed, the results can be converted back to the more common form of cylinder and axis (polar coordinates) for the results to be meaningful to the clinician.

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