

articles

Calculating the surgically induced refractive change following ocular surgery

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ABSTRACT

Calculating the surgically induced refractive change following ocular surgery is important for evaluating the results of keratorefractive procedures, smaller incisions and various wound closures for cataract surgery, and the effect of suturing techniques and suture removal following corneal transplant surgery. We present a ten-step method of calculating the spherical- and cylindrical-induced refractive change in a manner suitable for a programmable calculator or personal computer. Several applications are given including (1) adding the overrefraction to the spectacle correction, (2) determining the surgically induced refractive change from the preoperative and postoperative refractions, (3) determining the surgically induced refractive change from the K-readings, (4) rotating axes, (5) determining the power at meridians oblique to the principal meridians of a spherocylinder, (6) determining the coupling ratio, and (7) averaging axes. Standard methods for calculating and reporting aggregate results are also given.

Key Words: calculating surgically induced corneal astigmatism, coupling ratio, cross cylinder solution, oblique cylinder

Calculating the surgically induced spherical and astigmatic change is required to evaluate existing and evolving corneal surgical techniques. A variety of formulas to calculate corneal astigmatic changes have been developed. All represent gross simplifications of the complex topographical changes that occur on the aspheric corneal surface. Nevertheless, these formulas provide extremely useful data that assist in planning and evaluating corneal and limbal surgery.

The classic formula for calculating the surgically induced refractive change (SIRC) was described over 100 years ago.¹ It assumes that the induced corneal spherical and astigmatic change can be represented by a sphere and cylinder that, when placed in front of the eye at the corneal vertex,

would produce the same optical effect as the surgery.² Some 125 years later, Jaffe and Clayman described three trigonometric methods for performing these calculations.³ The fundamental advantage of this approach is its inherent consistency between refractive and keratometric changes and its sound mathematical basis.

Cravy adopted a different approach and developed a formula based solely upon keratometric values that elegantly facilitated the aggregate analysis of groups of patients.⁴ Cravy's formula has been widely used to calculate post-cataract surgery astigmatic changes, as it permits the calculation of aggregate with-the-rule or against-the-rule changes. To address some of the inconsistencies in Cravy's formula, Koch and Russell developed a for-

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mula that was derived from his method (T. Russell, "A New Formula for Calculating Changes in Corneal Astigmatism," Symposium on Cataract, IOL, and Refractive Surgery, Boston, April 1991).

Although each of these formulas has unique merits, we believe it is important to establish a standardized method for calculating and reporting spherical and astigmatic changes. We therefore propose that a straightforward, ten-step method based on the oblique cross-cylinder solution be adopted as this standard, because it is based on measurable optical changes and integrates refractive and keratometric changes. We will present the mathematical basis of this method and its clinical applications, and give several examples that calculate the entire SIRC following ocular surgery. Adopting these techniques will result in uniform, consistent, and understandable reporting of results by different investigators.

METHODS

Mathematical Solution to Obliquely Crossed Spherocylinders

There are four published methods for obliquely crossed spherocylinders: rectangular coordinate method, graphic vector method, matrix method, and the law of sines and cosines method for the cylinder change.^{3,5} Each of these methods accomplishes the same task in a different way, using trigonometric identities which yield the same unique result for any single pair of obliquely crossed spherocylinders. We will present a modification of the rectangular coordinate method in ten steps, which is simple to perform by hand or to implement on a programmable calculator or computer.

Determine the resultant Spherocylinder 3 from obliquely crossed cylinders Spherocylinder 1 and Spherocylinder 2.

Given: Spherocylinder 1 = SC1
 where S1 = sphere of SC1
 C1 = cylinder of SC1
 A1 = axis of cylinder C1

Spherocylinder 2 = SC2
 where S2 = sphere of SC2
 C2 = cylinder of SC2
 A2 = axis of cylinder C2

Find: Spherocylinder 3 = SC3
 where S3 = sphere of SC3
 C3 = cylinder of SC3
 A3 = axis of cylinder C3

Step 1. Transpose SC1 and SC2 so their cylinders have the same sign.

Step 2. SC1 must be chosen so the value of A1 is smaller than A2.

Step 3. Find angle α , the difference between A2 and A1.

$$\alpha = A2 - A1$$

Note: α must be positive.

Step 4. Find angle 2β from the formula:

$$\tan 2\beta = \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha}$$

The denominator can sometimes be zero, which "blows up" on most computers that cannot divide by zero. The whole term actually approaches infinity when the denominator approaches zero, which results in 2β equaling 90° . A simple programming solution is to add a very small value to the denominator such as 0.0000000001.

Step 5. Find angle θ from the following formula:

$$\theta = \frac{(2\beta + 180^\circ)}{2}$$

Step 6. Determine the sphere contributed by the two cross cylinders (SC) from

$$SC = C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta)$$

Step 7. Determine the total spherical result (S3) from the following formula:

$$S3 = S1 + S2 + SC$$

Step 8. Determine the total cylindrical result (C3) from the following formula:

$$C3 = C1 + C2 - 2 SC$$

Step 9. Determine the resultant axis (A3) in standard notation from the following formula:

$$A3 = A1 + \theta$$

If A3 is greater than 180° , subtract 180° for standard axis notation; if A3 is negative, add 180° .

Step 10. Any spherocylinder (SC3) can be written in one of three forms: the plus or minus spherocylinder form and the cross cylinder form. The alternate spherocylindrical form (SC4) and cross cylinder form (XC5) are calculated in the following manner using the transposition rules.

A. Alternate spherocylindrical form (SC4) of SC3:

$$S4 = S3 + C3 = \text{sphere of SC4}$$

$$C4 = -C3 = \text{cylinder of SC4}$$

$$A4 = A3 \pm 90^\circ = \text{axis of SC4}$$

B. Cross cylinder form (XC5) of SC3:

$$C5A = S3 = \text{cross cylinder A of XC5}$$

$$A5A = A3 \pm 90^\circ = \text{axis of cross cylinder A of XC5}$$

$$C5B = S3 + C3 = \text{cross cylinder B of XC5}$$

$$A5B = A3 = \text{axis of cross cylinder B of XC5}$$

Although each of these three forms represents the exact same spherocylinder, we will

see that each form has specific benefits in visualizing the effect of the surgical procedure being evaluated.

I. APPLICATION 1. Adding over-refraction to spectacles.

Example 1A. Write the patient's final spectacle prescription given the spectacle Rx and the over-refraction, where

Equation 1: Final Rx = Old Rx + Over-refraction

Given: Old spectacle Rx: sphere = 10.00 D
cylinder = +2.00 D
axis = 180°

Over-refraction: sphere = +2.25 D
cylinder = +1.25 D
axis = 45°

STEP 1. Both spherocylinders are already in plus cylinder form.

STEP 2. The over-refraction has the smallest axis and is therefore chosen as SC1.

Over-refraction	Old Rx
S1 = +2.25	S2 = +10.00
C1 = +1.25	C2 = +2.00
A1 = 45°	A2 = 180°

STEP 3. $\alpha = A2 - A1 = 180^\circ - 45^\circ$
 $\alpha = 135^\circ$

STEP 4.

$$\tan 2\beta = \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha}$$

$$\tan 2\beta = \frac{+2 * \sin(270^\circ)}{+1.25 + 2 * \cos(270^\circ)} = \frac{+2 * (-1)}{+1.25 + 2 * (0)}$$

$$\tan 2\beta = \frac{-2}{+1.25} = -1.60$$

$$2\beta = -58^\circ$$

STEP 5.

$$\theta = \frac{(2\beta + 180^\circ)}{2}$$

$$\theta = \frac{(-58 + 180^\circ)}{2} = \frac{122^\circ}{2} = 61^\circ$$

STEP 6. $SC = C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta)$
 $= 1.25 * (\sin 61^\circ)^2 + 2 * [\sin(135^\circ - 61^\circ)]^2$
 $= 1.25 * (.875)^2 + 2 * (.961)^2$
 $= .956 + 1.85$

$$SC = +2.80$$

STEP 7. $S3 = S1 + S2 + SC$
 $= +2.25 + 10.00 + 2.80$

$$S3 = +15.05$$

STEP 8. $C3 = C1 + C2 - 2SC$
 $= +1.25 + 2.00 - 2 * (+2.80)$

$$C3 = -2.36$$

STEP 9. $A3 = A1 + \theta$
 $= 45^\circ + 61^\circ$

$$A3 = 106^\circ$$

STEP 10.

A. Alternate spherocylindrical form (SC4):
 $S4 = S3 + C3 = +15.05 + (-2.36) = +12.70$
 $C4 = -C3 = -(-2.36) = +2.36$
 $A4 = A3 \pm 90^\circ = 106^\circ - 90^\circ = 16^\circ$

B. Cross cylinder form (XC5):
 $C5A = S3 = +15.05$
 $A5A = A3 \pm 90^\circ = 106^\circ - 90^\circ = 16^\circ$
 $C5B = S3 + C3 = +15.05 + (-2.36) = +12.70$
 $A5B = A3 = 106^\circ$

The patient's final prescription written in the three standard axis forms would be

Plus cyl form : +12.70 + 2.36 × 16°

Minus cyl form : +15.05 - 2.36 × 106°

Cross cyl form : +15.05 × 16° and +12.70 × 106°

This same method can be used for refining toric soft contact lens prescriptions using an over-refraction.⁶

II. APPLICATION 2. Determining the SIRC from the preoperative refraction (Rpre) and the postoperative refraction (Rpost).

Before applying the obliquely crossed cylinder solution we must review some basic principles of refraction. First, the basic principle is that the error of the eye (EE) plus the optical correction (Rx) must equal the residual error (RE). Expressed algebraically this would be

$$\text{Equation A: } EE + Rx = RE$$

If there is no residual error, i.e., the optical correction is exactly correct, then RE is 0, and the error of the eye is exactly equal to the negative of the optical correction.

$$\text{Equation B: } EE = -Rx, \text{ when } RE = 0$$

For spherical refractive errors this is readily apparent. If a patient is myopic by 1 diopter (D), the optical prescription is -1.00 D and the error of the eye is +1.00 D. The plus sign simply indicates that the eye has an excess power of 1.00 D. This same relationship holds true for spherocylindrical refractive errors. One needs only to change the signs of the optical correction to determine the error of the eye. For example,

$$\begin{aligned} \text{If } Rx &= -1.00 + 1.00 \times 90^\circ \\ \text{then } EE &= +1.00 - 1.00 \times 90^\circ. \end{aligned}$$

Only the signs change; the axis remains the same. Now, using equation A,

$$EE + Rx = RE$$

we can let EE be the preoperative error of the eye and RE be the residual error of the eye after surgery. Rx is the "correction" or the SIRC that has

been added to the preoperative error of the eye to achieve the postoperative residual error. Solving equation A for Rx, we get

$$Rx = RE - EE$$

or the SIRC is the residual error of the eye after surgery minus the original error of the eye.

$$\text{Equation C: SIRC} = RE - EE$$

Since we are more accustomed to working with the refractions than the error of the eye, let us rewrite equation C using the refractions.

$$\begin{aligned} \text{Preoperative} &= \text{Negative error} \\ \text{refraction} &= \text{of the eye preoperatively} \\ \text{PreRx} &= -EE \end{aligned}$$

$$\begin{aligned} \text{Postoperative} &= \text{Negative error} \\ \text{refraction} &= \text{of the eye postoperatively} \\ \text{PostRx} &= -RE \end{aligned}$$

Substituting in Equation C,

$$\text{SIRC} = -\text{PostRx} + \text{PreRx}$$

or

$$\text{Equation 2: SIRC} = \text{PreRx} - \text{PostRx}$$

Thus, the SIRC can be determined by adding the preoperative refraction to the negative of the postoperative refraction. Note that if SIRC is plus, power has been added to the eye (myopic direction) and, if SIRC is negative, the power of the eye has been reduced (hyperopic direction).

Example 2A. A patient has radial keratotomy with a preoperative refraction (PreRx) of -4.00 D and postoperatively (PostRx) has a refraction of $+0.50$ D. What was the SIRC?

$$\begin{aligned} \text{PreRx} &= -4.00 \text{ D} \\ \text{PostRx} &= +0.50 \text{ D} \\ \text{SIRC} &= \text{PreRx} - \text{PostRx} \\ &= -4.00 - (+0.50) \\ \text{SIRC} &= -4.50 \text{ D} \end{aligned}$$

The surgery has reduced the power of the eye by 4.50 D and has the same effect as having placed a -4.50 D sphere in front of the patient preoperatively.

Example 2B. A patient has radial and astigmatic keratotomy with a preoperative refraction of $-5.25 + 1.00 \times 80^\circ$ and postoperatively has a refraction of $-0.50 + 0.75 \times 135^\circ$. What was the SIRC?

$$\begin{aligned} \text{PreRx} &= -5.25 + 1.00 \times 80^\circ \\ \text{PostRx} &= -0.50 + 0.75 \times 135^\circ \\ -\text{PostRx} &= +0.50 - 0.75 \times 135^\circ \end{aligned}$$

Now we must apply the ten steps for the obliquely crossed cylinder solution to PreRx and $-\text{PostRx}$, where

$$\text{SIRC} = \text{PreRx} - \text{PostRx}$$

STEP 1. Transpose one of the spherocylinders so the cylinders have the same sign.

$$-\text{PostRx} = -0.25 + 0.75 \times 45^\circ$$

STEP 2. $-\text{PostRx}$ should be chosen as spherocylinder 1 (SC1) since it has the smaller angle.

$-\text{PostRx}$	PreRx
S1 = -0.25	S2 = -5.25
C1 = $+0.75$	C2 = $+1.00$
A1 = 45°	A2 = 80°

STEP 3. $\alpha = A2 - A1 = 80^\circ - 45^\circ$

$$\alpha = 35^\circ$$

STEP 4.

$$\begin{aligned} \text{Tan } 2\beta &= \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha} \\ \text{Tan } 2\beta &= \frac{+1 * \sin(70^\circ)}{+0.75 + 1 * \cos(70^\circ)} = \frac{+1 * (.940)}{+0.75 + 1 * (.342)} \\ &= 0.861 \\ 2\beta &= 40.7^\circ \end{aligned}$$

STEP 5.

$$\begin{aligned} \theta &= \frac{(2\beta + 180^\circ)}{2} \\ \theta &= \frac{(40.7^\circ + 180^\circ)}{2} = \frac{220.7^\circ}{2} = 110.4^\circ \end{aligned}$$

$$\begin{aligned} \text{STEP 6. SC} &= C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta) \\ &= 0.75 * (\sin 110.4^\circ)^2 + 1 * [\sin(35^\circ - 110.4^\circ)]^2 \\ &= 0.75 * (.938)^2 + 1 * (-.968)^2 \\ &= .659 + 0.936 \\ \text{SC} &= +1.60 \end{aligned}$$

$$\begin{aligned} \text{STEP 7. S3} &= S1 + S2 + \text{SC} \\ &= -0.25 + (-5.25) + 1.60 \\ \text{S3} &= -3.90 \end{aligned}$$

$$\begin{aligned} \text{STEP 8. C3} &= C1 + C2 - 2\text{SC} \\ &= +0.75 + 1.00 - 2 * (+1.60) \\ \text{C3} &= -1.44 \end{aligned}$$

$$\begin{aligned} \text{STEP 9. A3} &= A1 + \theta \\ &= 45^\circ + 110.4^\circ \\ \text{A3} &= 155^\circ \end{aligned}$$

STEP 10.

A. Alternate spherocylindrical form (SC4):

$$\begin{aligned} \text{S4} &= \text{S3} + \text{C3} = -3.90 + (-1.44) = -5.34 \\ \text{C4} &= -\text{C3} = -(-1.44) = +1.44 \\ \text{A4} &= \text{A3} \pm 90^\circ = 155^\circ - 90^\circ = 65^\circ \end{aligned}$$

B. Cross cylinder form (XC5):

$$\begin{aligned} \text{C5A} &= \text{S3} = -3.90 \\ \text{A5A} &= \text{A3} \pm 90^\circ = 155^\circ - 90^\circ = 65^\circ \\ \text{C5B} &= \text{S3} + \text{C3} = -3.90 + (-1.44) = -5.34 \\ \text{A5B} &= \text{A3} = 155^\circ \end{aligned}$$

The patient's SIRC written in the three standard axis forms would be

$$\begin{aligned} \text{Plus cyl form} &: -5.34 + 1.44 \times 65^\circ \\ \text{Minus cyl form} &: -3.90 - 1.44 \times 155^\circ \\ \text{Cross cyl form} &: -3.90 \times 65^\circ \text{ and } -5.34 \times 155^\circ \end{aligned}$$

Since radial keratotomy causes a reduction in corneal power, the minus cylinder form is the easiest to visualize and interpret. The radial and astigmatic keratotomy has caused a spherical reduction in power of 3.90 D (in all meridians) and

an additional 1.44 D of reduction at an axis of 155°. Remember that our refractive change has always been in axis notation and is equivalent to placing this refractive change in front of the patient preoperatively.

When dealing with surgery, it is usually easier to see what is happening by expressing the resulting refractive change in its *power form*, i.e.,

$$+1.00 \times 90^\circ = +1.00 @ 180^\circ$$

The “@” symbol is used to designate power notation.

Expressing this patient’s actual SIRC in the three *power notation forms*, we have

- Plus cyl form : $-5.34 + 1.44 @ 155^\circ$
- Minus cyl form : $-3.90 - 1.44 @ 65^\circ$
- Cross cyl form : $-3.90 @ 155^\circ$ and $-5.34 @ 65^\circ$

Again, in the minus cylinder power form, we see that the cornea has been flattened by 3.90 D in all meridians and has been flattened by an additional 1.44 D at the 65° meridian. The power notation in the cross cylinder form shows the total flattening in each of the principal corneal meridians, i.e., 3.90 D @ 155° and 5.34 D @ 65°.

The perfect result would have occurred if the SIRC were exactly equal to the preoperative refraction. The preoperative prescription was $-5.25 + 1.00 \times 80^\circ$. In minus cylinder form this would be $-4.25 - 1.00 \times 170^\circ$. Expressed in the power notation, we would have

$$-4.25 - 1.00 \times 170^\circ = -4.25 - 1.00 @ 80^\circ$$

$$\text{Perfect SIRC} = -4.25 @ 170^\circ \text{ and } -5.25 @ 80^\circ$$

We see that the postoperative residual refractive error is primarily because the surgical effect is 15° off the meridians in a clockwise direction for the perfect surgical result. The principal meridians of the surgical effect were at 155° and 65°, when they should have been at 170° and 80°. This finding would have been almost impossible to determine intuitively.

Example 2C. A patient has a cataract operation with intraocular lens (IOL) implantation. The preoperative refraction is $-1.00 + 1.00 \times 80^\circ$. One week later the postoperative refraction is $-4.00 + 3.00 \times 90^\circ$. What was the SIRC?

$$\begin{aligned} \text{PreRx} &= -1.00 + 1.00 \times 80^\circ \\ \text{PostRx} &= -4.00 + 3.00 \times 90^\circ \\ \text{-PostRx} &= +4.00 - 3.00 \times 90^\circ \end{aligned}$$

Now we must apply the ten steps for the obliquely crossed cylinder solution to PreRx and -PostRx for SIRC.

STEP 1. Transpose one of the spherocylinders so the cylinders have the same sign.

$$\text{-PostRx} = +1.00 + 3.00 \times 180^\circ$$

STEP 2. PreRx should be chosen as spherocylinder 1 (SC1) since it has the smaller angle.

<i>PreRx</i>	<i>-PostRx</i>
S1 = -1.00	S2 = +1.00
C1 = +1.00	C2 = +3.00
A1 = 80°	A2 = 180°

STEP 3. $\alpha = A2 - A1 = 180^\circ - 80^\circ$
 $\alpha = 100^\circ$

STEP 4.

$$\tan 2\beta = \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha}$$

$$\tan 2\beta = \frac{+3 * \sin(200^\circ)}{+1 + 3 * \cos(200^\circ)} = \frac{+3 * (-.342)}{+1.00 + 3 * (-.940)}$$

$$= 0.564$$

$$2\beta = 29.4^\circ$$

STEP 5.

$$\theta = \frac{(2\beta + 180^\circ)}{2}$$

$$\theta = \frac{(29.4^\circ + 180^\circ)}{2} = \frac{209.4^\circ}{2} = 104.7^\circ$$

STEP 6. $SC = C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta)$
 $= 1.00 * (\sin 104.7^\circ)^2 + 3 * [\sin(100^\circ - 104.7^\circ)]^2$
 $= 1.00 * (.967)^2 + 3 * (-0.082)^2$
 $= .935 + .020$
 $SC = 0.956 = 0.96$

STEP 7. $S3 = S1 + S2 + SC$
 $= -1.00 + (1.00) + 0.956$
 $S3 = +0.96$

STEP 8. $C3 = C1 + C2 - 2SC$
 $= +1.00 + 3.00 - 2 * (+0.956)$
 $C3 = +2.09$

STEP 9. $A3 = A1 + \theta$
 $= 80^\circ + 104.7^\circ$
 $A3 = 184.7^\circ = 4.7^\circ = 5^\circ$

STEP 10.

- A. Alternate spherocylindrical form (SC4):
 $S4 = S3 + C3 = +0.956 + (+2.09) = +3.05$
 $C4 = -C3 = -(+2.09) = -2.09$
 $A4 = A3 \pm 90^\circ = 5^\circ + 90^\circ = 95^\circ$
- B. Cross cylinder form (XC5):
 $C5A = S3 = +0.96$
 $A5A = A3 \pm 90^\circ = 5^\circ + 90^\circ = 95^\circ$
 $C5B = S3 + C3 = +0.96 + (+2.09) = +3.05$
 $A5B = A3 = 5^\circ$

The patient’s SIRC written in the three standard *axis forms* would be

$$\begin{aligned} \text{Plus cyl form} &: +0.96 + 2.09 \times 5^\circ \\ \text{Minus cyl form} &: +3.05 - 2.09 \times 95^\circ \\ \text{Cross cyl form} &: +0.96 \times 95^\circ \text{ and } +3.05 \times 5^\circ \end{aligned}$$

Expressing this patient’s actual SIRC in power notation, we have

$$\begin{aligned} \text{Plus cyl form} &: +0.96 + 2.09 @ 95^\circ \\ \text{Minus cyl form} &: +3.05 - 2.09 @ 5^\circ \\ \text{Cross cyl form} &: +0.96 @ 5^\circ \text{ and } +3.05 @ 95^\circ \end{aligned}$$

In this case, the plus cylinder power form best describes the surgical effect. The power of the eye

increases by a spherical component of +0.96 D and by a cylindric component of +2.09 @ 95°. This change is due to the change in the cornea and the power difference between the implanted IOL and the patient's cataractous crystalline lens prior to surgery. The exact change due to the cornea can only be determined by the change in the K-readings as shown for this patient in example 3C.

The cylindric change calculated from the refractions does not correspond to the change calculated from the K-readings for one of two reasons: (1) the corneal astigmatism is irregular and keratometer readings are inaccurate or (2) the IOL has induced an astigmatic component. This can occur from a poorly manufactured IOL with astigmatism (rare) or from tilt of a spherical lens relative to the optical axis of the eye. If the lens is tilted and the power of the lens is known, the exact angle of the lens tilt can be determined.⁷

Example 2D. One year later, patient in example 2C, who had a cataract operation and IOL implantation with a preoperative refraction of -1.00 + 1.00 × 80°, had a postoperative refraction of plano - 1.00 × 90°. What was the SIRC?

$$\begin{aligned} \text{PreRx} &= -1.00 + 1.00 \times 80^\circ \\ \text{PostRx} &= 0.00 - 1.00 \times 90^\circ \\ -\text{PostRx} &= 0.00 + 1.00 \times 90^\circ \end{aligned}$$

Now we must apply the ten steps for the obliquely crossed cylinder solution to PreRx and -PostRx for SIRC.

STEP 1. The spherocylinders are already in the plus cylinder form.

$$-\text{PostRx} = 0.00 + 1.00 \times 90^\circ$$

STEP 2. PreRx should be chosen as spherocylinder 1 (SC1) since it has the smaller angle.

<i>PreRx</i>	<i>-PostRx</i>
S1 = -1.00	S2 = 0.00
C1 = +1.00	C2 = +1.00
A1 = 80°	A2 = 90°

STEP 3. $\alpha = A2 - A1 = 90^\circ - 80^\circ$

$$\alpha = 10^\circ$$

STEP 4.

$$\begin{aligned} \tan 2\beta &= \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha} \\ \tan 2\beta &= \frac{+1 * \sin(20^\circ)}{+1 + 1 * \cos(20^\circ)} = \frac{+1 * (.342)}{+1.00 + 1 * (.940)} \\ &= 0.176 \\ 2\beta &= 10^\circ \end{aligned}$$

STEP 5.

$$\begin{aligned} \theta &= \frac{(2\beta + 180^\circ)}{2} \\ \theta &= \frac{(10^\circ + 180^\circ)}{2} = \frac{190^\circ}{2} = 95^\circ \end{aligned}$$

$$\begin{aligned} \text{STEP 6. SC} &= C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta) \\ &= 1.00 * (\sin 95^\circ)^2 + 1 * [\sin(10^\circ - 95^\circ)]^2 \\ &= 1.00 * (.996)^2 + 1 * (-.996)^2 \\ &= .992 + .992 \end{aligned}$$

$$\text{SC} = 1.985$$

$$\begin{aligned} \text{STEP 7. S3} &= S1 + S2 + \text{SC} \\ &= -1.00 + (0.00) + 1.985 \\ \text{S3} &= +0.985 \end{aligned}$$

$$\begin{aligned} \text{STEP 8. C3} &= C1 + C2 - 2\text{SC} \\ &= +1.00 + 1.00 - 2 * (1.985) \\ \text{C3} &= -1.970 \end{aligned}$$

$$\begin{aligned} \text{STEP 9. A3} &= A1 + \theta \\ &= 80^\circ + 95^\circ \\ \text{A3} &= 175^\circ \end{aligned}$$

STEP 10.

A. Alternate spherocylindrical form (SC4):

$$\text{S4} = \text{S3} + \text{C3} = +0.985 + (-1.970) = -0.985$$

$$\text{C4} = -\text{C3} = -(-1.970) = +1.970$$

$$\text{A4} = \text{A3} \pm 90^\circ = 175^\circ - 90^\circ = 85^\circ$$

B. Cross cylinder form (XC5):

$$\text{C5A} = \text{S3} = +0.985$$

$$\text{A5A} = \text{A3} \pm 90^\circ = 175^\circ - 90^\circ = 85^\circ$$

$$\text{C5B} = \text{S3} + \text{C3} = +0.985 + (-1.970) = -0.985$$

$$\text{A5B} = \text{A3} = 175^\circ$$

The patient's SIRC written in the three axis forms would be

$$\begin{aligned} \text{Plus cyl form} &: -0.98 + 1.97 \times 85^\circ \\ \text{Minus cyl form} &: +0.98 - 1.97 \times 175^\circ \\ \text{Cross cyl form} &: +0.98 \times 85^\circ \text{ and } -0.98 \times 175^\circ \end{aligned}$$

Expressing this patient's actual SIRC in three power forms, we have

$$\begin{aligned} \text{Plus cyl form} &: -0.98 + 1.97 @ 175^\circ \\ \text{Minus cyl form} &: +0.98 - 1.97 @ 85^\circ \\ \text{Cross cyl form} &: +0.98 @ 175^\circ \text{ and } -0.98 @ 85^\circ \end{aligned}$$

In this example, the minus cylinder form provides the best visualization of the surgical result. The surgeon has caused 1.97 D of power reduction in the meridian @ 85°. In short, the wound has "faded" (flattened) approximately 2.0 D in the near vertical meridian. The spherical power change is due not only to the corneal change, but also to the difference between the power of the IOL and the patient's crystalline lens before surgery. The exact spherical change in the cornea can be calculated only by using the K-readings as shown in example 3D.

III. APPLICATION 3. Determining the SIRC from the preoperative K-readings (Kpre) and the postoperative K-readings (Kpost).

Example 3A. A patient has radial keratotomy with preoperative spherical K-readings (Kpre) of 44.00 D @ 90° and @ 180° and postoperatively has K-readings of 39.50 D @ 90° and @ 180°. What was the SIRC?

K-readings are already in power notation as demonstrated by the @ symbol. Note that each K-reading should have an associated meridian, not just one of the two K-readings. The relationship for this application is the preoperative K-readings plus the SIRC is equal to the postoperative K-readings.

$$K_{pre} + SIRC = K_{post}$$

Equation 3: $SIRC = K_{post} - K_{pre}$

$$\begin{aligned} K_{pre} &= +44.00 \text{ D} \\ -K_{pre} &= -44.00 \text{ D} \\ K_{post} &= +39.50 \text{ D} \\ SIRC &= K_{post} - K_{pre} \\ &= +39.50 \text{ D} - 44.00 \\ SIRC &= -4.50 \text{ D} \end{aligned}$$

The surgery has reduced the power of the eye by 4.50 D and has the same effect as having placed a -4.50 D sphere in front of the patient preoperatively.

Example 3B. The patient in example 2B has radial and astigmatic keratotomy with preoperative K-readings of 44.00 D @ 80° and 43.00 D @ 170° and postoperatively has K-readings of 38.50 D @ 45° and 39.25 D @ 135°. What was the SIRC?

$$\begin{aligned} K_{post} &= 38.50 @ 45^\circ \text{ and } 39.25 @ 135^\circ = 38.50 + 0.75 @ 135^\circ \\ K_{pre} &= 44.00 @ 80^\circ \text{ and } 43.00 @ 170^\circ = 44.00 - 1.00 @ 170^\circ \\ -K_{pre} &= -44.00 + 1.00 @ 170^\circ \end{aligned}$$

Now we must apply the ten steps for the obliquely crossed cylinder solution to K_{post} and $-K_{pre}$ for SIRC from equation 3.

- STEP 1. Both K_{post} and $-K_{pre}$ already have the same sign in the plus cylinder form.
STEP 2. K_{post} should be chosen as spherocylinder 1 (SC1) since it has the smaller angle.

K_{post}	$-K_{pre}$
$S1 = +38.50$	$S2 = -44.00$
$C1 = +0.75$	$C2 = +1.00$
$A1 = 135^\circ$	$A2 = 170^\circ$

STEP 3. $\alpha = A2 - A1 = 170^\circ - 135^\circ$
 $\alpha = 35^\circ$

STEP 4.

$$\begin{aligned} \tan 2\beta &= \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha} \\ \tan 2\beta &= \frac{+1 * \sin(70^\circ)}{+0.75 + 1 * \cos(70^\circ)} = \frac{+1 * (.940)}{+0.75 + 1 * (.342)} \\ &= 0.861 \\ 2\beta &= 40.7^\circ \end{aligned}$$

STEP 5.

$$\begin{aligned} \theta &= \frac{(2\beta + 180^\circ)}{2} \\ \theta &= \frac{(40.7^\circ + 180^\circ)}{2} = \frac{220.7^\circ}{2} = 110.4^\circ \end{aligned}$$

STEP 6. $SC = C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta)$
 $= 0.75 * (\sin 110.4^\circ)^2 + 1 * [\sin(35^\circ - 110.4^\circ)]^2$
 $= 0.75 * (.938)^2 + 1 * (-.968)^2$
 $= .659 + 0.936$
 $SC = +1.60$

STEP 7. $S3 = S1 + S2 + SC$
 $= +38.50 + (-44.00) + 1.60$
 $S3 = -3.90$
STEP 8. $C3 = C1 + C2 - 2SC$
 $= +0.75 + 1.00 - 2 * (+1.60)$
 $C3 = -1.44$
STEP 9. $A3 = A1 + \theta$
 $= 135^\circ + 110.4^\circ$
 $A3 = 245^\circ \text{ or } 65^\circ (245^\circ - 180^\circ)$
 $A3 = 65^\circ$

STEP 10.

- A. Alternate spherocylindrical form (SC4):
 $S4 = S3 + C3 = -3.90 + (-1.44) = -5.34$
 $C4 = -C3 = -(-1.44) = +1.44$
 $A4 = A3 \pm 90^\circ = 65^\circ + 90^\circ = 155^\circ$
B. Cross cylinder form (XC5):
 $C5A = S3 = -3.90$
 $A5A = A3 \pm 90^\circ = 65^\circ + 90^\circ = 155^\circ$
 $C5B = S3 + C3 = -3.90 + (-1.44) = -5.34$
 $A5B = A3 = 65^\circ$

The patient's SIRC written in the three power forms would be

Plus cyl form : $-5.34 + 1.44 @ 155^\circ$
Minus cyl form : $-3.90 - 1.44 @ 65^\circ$
Cross cyl form : $-3.90 @ 155^\circ$ and $-5.34 @ 65^\circ$

These results agree exactly with what we found by refraction in example 2B, as it should. In the real world, however, this rarely occurs following keratorefractive surgery because the K-readings are not accurate in these corneas, and the endpoint of the refraction may not be precise. Usually the calculations using the refraction are more useful, particularly for writing the spectacle correction, but as our ability to accurately measure corneal power improves, the results will become closer.

Example 3C. The patient in example 2C, who had a cataract operation with IOL implantation, had preoperative K-readings of 44.00 D @ 80° and 43.00 D @ 170°. One week postoperatively the K-readings were 43.00 D @ 180° and 46.00 D @ 90°. What was the SIRC?

$$\begin{aligned} K_{post} &= 43.00 @ 180^\circ \text{ and } 46.00 @ 90^\circ \\ K_{post} &= 43.00 + 3.00 @ 90^\circ \\ K_{pre} &= 44.00 @ 80^\circ \text{ and } 43.00 @ 170^\circ \\ K_{pre} &= 44.00 - 1.00 @ 170^\circ \\ -K_{pre} &= -44.00 + 1.00 @ 170^\circ \end{aligned}$$

Now we must apply the ten steps for the obliquely crossed cylinder solution to K_{post} and $-K_{pre}$ for SIRC from equation 3.

- STEP 1. Both K_{post} and $-K_{pre}$ already have the same sign in the plus cylinder form.
STEP 2. K_{post} should be chosen as spherocylinder 1 (SC1) since it has the smaller angle.

K_{post}	$-K_{pre}$
$S1 = 43.00$	$S2 = -44.00$
$C1 = +3.00$	$C2 = +1.00$
$A1 = 90^\circ$	$A2 = 170^\circ$

STEP 3. $\alpha = A2 - A1 = 170^\circ - 90^\circ$
 $\alpha = 80^\circ$

STEP 4.

$$\tan 2\beta = \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha}$$

$$\tan 2\beta = \frac{+1 * \sin(160^\circ)}{+3 + 1 * \cos(160^\circ)} = \frac{+1 * (.342)}{+3.00 + 1 * (-.940)}$$

$$= 0.166$$

$$2\beta = 9.43^\circ$$

STEP 5.

$$\theta = \frac{(2\beta + 180^\circ)}{2}$$

$$\theta = \frac{(9.43^\circ + 180^\circ)}{2} = \frac{189.43^\circ}{2} = 94.7^\circ$$

STEP 6. $SC = C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta)$
 $= 3.00 * (\sin 94.7^\circ)^2 + 1 * [\sin(80^\circ - 94.7^\circ)]^2$
 $= 3.00 * (.997)^2 + 1 * (-.254)^2$
 $= 2.98 + .065$

$$SC = 3.044 = 3.04$$

STEP 7. $S3 = S1 + S2 + SC$
 $= +43.00 + (-44.00) + 3.04$
 $S3 = +2.04$

STEP 8. $C3 = C1 + C2 - 2SC$
 $= +3.00 + 1.00 - 2 * (3.044)$
 $C3 = -2.09$

STEP 9. $A3 = A1 + \theta$
 $= 90^\circ + 94.7^\circ$
 $A3 = 184.7^\circ \text{ or } 4.7^\circ = 5^\circ$

STEP 10.

A. Alternate spherocylindrical form (SC4):

$$S4 = S3 + C3 = +2.044 + (-2.088) = -0.04$$

$$C4 = -C3 = -(-2.09) = +2.09$$

$$A4 = A3 \pm 90^\circ = 5^\circ + 90^\circ = 95^\circ$$

B. Cross cylinder form (XC5):

$$C5A = S3 = +2.04$$

$$A5A = A3 \pm 90^\circ = 5^\circ + 90^\circ = 95^\circ$$

$$C5B = S3 + C3 = +2.044 + (-2.088) = -0.04$$

$$A5B = A3 = 5^\circ$$

The patient's SIRC written in the three power forms would be

$$\text{Plus cyl form : } -0.04 + 2.09 @ 95^\circ$$

$$\text{Minus cyl form : } +2.04 - 2.09 @ 5^\circ$$

$$\text{Cross cyl form : } +2.04 @ 95^\circ \text{ and } -0.04 @ 5^\circ$$

In this case, the plus cylinder power form shows that there was almost no change in corneal power 90° away from the surgical meridian, but +2.09 D of increased power @ 95° occurred. The 95° meridian is the central location of the tight suture force. The spherical equivalent of the patient's SIRC due to the cornea is +1.00 D. From example 2C, the spherical equivalent change of the entire eye was +2.00 D. Therefore, the IOL power is +1 D stronger than the patient's cataractous crystalline lens.

Example 3D. The patient in example 3C had a cataract operation with IOL implantation. Preoperative K-readings were 44.00 D @ 80° and 43.00 D @ 170°; one year post-operatively K-readings were 43.00 D @ 180° and 42.00 D

@ 90°. What was the SIRC?

$$K_{\text{post}} = 43.00 @ 180^\circ \text{ and } 42.00 @ 90^\circ = 42.00 + 1.00 @ 180^\circ$$

$$K_{\text{pre}} = 44.00 @ 80^\circ \text{ and } 43.00 @ 170^\circ = 44.00 - 1.00 @ 170^\circ$$

$$-K_{\text{pre}} = -44.00 + 1.00 @ 170^\circ$$

Now we must apply the ten steps for the obliquely crossed cylinder solution to K_{post} and $-K_{\text{pre}}$ for SIRC from equation 3.

STEP 1. Both K_{post} and $-K_{\text{pre}}$ have the same sign in the plus cylinder form.

STEP 2. K_{pre} should be chosen as spherocylinder 1 (SC1) since it has the smaller angle.

$-K_{\text{pre}}$	K_{post}
$S1 = -44.00$	$S2 = 42.00$
$C1 = +1.00$	$C2 = +1.00$
$A1 = 170^\circ$	$A2 = 180^\circ$

STEP 3. $\alpha = A2 - A1 = 180^\circ - 170^\circ$

$$\alpha = 10^\circ$$

STEP 4.

$$\tan 2\beta = \frac{C2 \sin 2\alpha}{C1 + C2 \cos 2\alpha}$$

$$\tan 2\beta = \frac{+1 * \sin(20^\circ)}{+1 + 1 * \cos(20^\circ)} = \frac{+1 * (.342)}{+1.00 + 1 * (.940)}$$

$$= 0.176$$

$$2\beta = 10^\circ$$

STEP 5.

$$\theta = \frac{(2\beta + 180^\circ)}{2}$$

$$\theta = \frac{(10^\circ + 180^\circ)}{2} = \frac{190^\circ}{2} = 95^\circ$$

STEP 6. $SC = C1 \sin^2 \theta + C2 \sin^2 (\alpha - \theta)$
 $= 1.00 * (\sin 95^\circ)^2 + 1 * [\sin(10^\circ - 95^\circ)]^2$
 $= 1.00 * (.996)^2 + 1 * (-.996)^2$
 $= .992 + .992$

$$SC = 1.985$$

STEP 7. $S3 = S1 + S2 + SC$
 $= -44.00 + (42.00) + 1.985$
 $S3 = -0.015$

STEP 8. $C3 = C1 + C2 - 2SC$
 $= +1.00 + 1.00 - 2 * (1.985)$
 $C3 = -1.970$

STEP 9. $A3 = A1 + \theta$
 $= 170^\circ + 95^\circ$
 $A3 = 85^\circ$

STEP 10.

A. Alternate spherocylindrical form (SC4):

$$S4 = S3 + C3 = -0.015 + (-1.970) = -1.985$$

$$C4 = -C3 = -(-1.970) = +1.970$$

$$A4 = A3 \pm 90^\circ = 85^\circ + 90^\circ = 175^\circ$$

B. Cross cylinder form (XC5):

$$C5A = S3 = -0.015$$

$$A5A = A3 \pm 90^\circ = 85^\circ + 90^\circ = 175^\circ$$

$$C5B = S3 + C3 = -0.015 + (-1.970) = -1.985$$

$$A5B = A3 = 85^\circ$$

The patient's SIRC written in the three power forms would be

Plus cyl form : $-1.98 + 1.97 @ 175^\circ$
 Minus cyl form : $-0.02 - 1.97 @ 85^\circ$
 Cross cyl form : $-0.02 @ 175^\circ$ and $-1.98 @ 85^\circ$

As we saw using the preoperative and postoperative refraction in example 2D, the minus cylinder form provided the best description of the surgical result since flattening occurred. The surgeon has caused 1.97 D of power reduction in the meridian @ 85° . In short, the wound has "faded" (flattened) approximately 2 D in the near vertical meridian. The exact change in the spherical component of the cornea was calculated as -0.02 D. The total $+0.98$ D, found by using the refraction, indicates that the IOL added $+1.00$ D sphere and was therefore 1 D stronger than the patient's cataractous crystalline lens.

IV. ROTATING AXES. Sometimes it is necessary to compare the SIRC in cases in which the same operation is performed but at different meridians of the eye. For example, the same cataract operation may be performed at 12 o'clock in one eye and temporally or at 9 o'clock in another eye. How much flattening occurs in the meridian of the incision?

1. Apply the ten steps for determining the SIRC for each case.
2. Determine the standard reference meridian for the typical surgery; e.g., cataract surgery is most often at 12 o'clock so the reference meridian would be @ 90° .
3. Record the actual meridian that was used at the time of surgery; e.g., a temporal approach in the right eye would be at 180° . With cylinders there is no way of determining a semimeridional effect because they are assumed to be equal. Because of this assumption, 90° and 270° are equal as are 0° and 180° . When calculations result in angles greater than 180° , simply subtract 180° to find the standard notation. If calculations result in angles that are negative, simply add 180° . Adding or subtracting 180° in this manner will not change the results for cylinders.
4. Determine the rotation angle R.

Equation 4: Rotation angle R = actual meridian of surgery - reference meridian

5. Subtract the rotation angle R from the axis of the SIRC determined for that case. All the results will appear to have incisions at the reference meridian.

Example: Suppose we wanted to determine the SIRC that results from T-cuts and the reference was chosen @ 90° .

Case 1. Incisions at 90° and the SIRC was $-1.00 + 2.00 \times 90^\circ$. Rotation angle R is zero—no change. Cross cylinder

form of SIRC is $-1.00 \times 180^\circ$ and $+1.00 \times 90^\circ$ or in the power notation $-1.00 @ 90^\circ$ and $+1.00 @ 180^\circ$.

Case 2. Incisions at 70° and the SIRC was $-1.00 + 2.00 \times 70^\circ$. Rotation angle R is -20° ($70^\circ - 90^\circ$). The rotated SIRC would be $-1.00 + 2.00 \times 90^\circ$.

Case 3. Incisions at 110° and the SIRC was $-1.00 + 2.00 \times 110^\circ$. Rotation angle R is $+20^\circ$ ($110^\circ - 90^\circ$). The rotated SIRC would be $-1.00 + 2.00 \times 90^\circ$.

Notice that the SIRC for each of these cases is identical when the site of the incision is corrected for the rotation angle R. Each case showed 1 D of flattening in the 90° meridian and 1 D of steepening in the 180° meridian.

V. DETERMINING THE POWER AT MERIDIANS OBLIQUE TO THE PRINCIPAL MERIDIANS. Occasionally, the contribution of a cylinder to a meridian oblique to the cylinder's axis may be needed.⁸ For example, calculating the power in the vertical and horizontal meridians for an oblique cylinder is necessary for determining the induced prism difference between the two lenses in a pair of spectacles. It can also be used to determine the contribution of a cylinder to the surgical meridian. The principle is based on the fact that the power of a cylinder changes by a cosine-squared function as it is rotated. The contribution of the cylinder to a meridian angle R degrees away from the axis of the cylinder is equal to the magnitude of the cylinder multiplied by the cosine-squared of angle R.

Example: Find the contribution of power to the vertical (90°) and horizontal (180°) meridians for the cylinder $+2.00 \times 60^\circ$.

Step 1. Convert the cylinder to the power notation.
 $+2.00 \times 60^\circ = +2.00 @ 150^\circ$

Step 2. Determine the angle R between the power meridian of the cylinder and meridian of interest.

Vertical meridian: $R = 150^\circ - 90^\circ$
 $R = +60^\circ$

Equation 5A:

Vertical component = cylinder power * $(\cos R)^2$

$= +2.00 \quad * [\cos(+60^\circ)]^2$
 $= +2.00 \quad * [0.50]^2$
 $= +2.00 \quad * [0.25]$

Vertical component = $+0.50$ D

Horizontal meridian: $R = 150^\circ - 180^\circ$
 $R = -30^\circ$

Equation 5B:

Horizontal component = cylinder power * $(\cos R)^2$

$= +2.00 \quad * [\cos(-30^\circ)]^2$
 $= +2.00 \quad * [0.866]^2$
 $= +2.00 \quad * [0.75]$

Horizontal component = $+1.50$ D

The result is that a cylinder of $+2.00 \times 60^\circ$ contributes $+0.50$ D of power to the vertical meridian and $+1.50$ D to the horizontal. Notice that the sum of the two components exactly equals the original cylinder power.

VI. DETERMINING THE COUPLING RATIO.

The "coupling effect" refers to the changes in power that occur in the surgical meridian and 90° away from the surgical meridian. The coupling ratio (CR) is the change in power in these two meridians, expressed as a ratio. Unfortunately, the definition is not standardized; thus some authors calculate the ratio by placing the change in the surgical meridian in the numerator and others, in the denominator.⁹

In general, a CR is defined as:

$$\text{CR} = \frac{\text{Reaction}}{\text{Action}} = \frac{\text{Secondary effect}}{\text{Primary effect}}$$

Similar to optical magnification, using the denominator as the primary effect serves as the reference to which the secondary effect (the reaction) is compared. Like optical magnification, values less than one indicate that the secondary effect is less than the primary effect, and values greater than one indicate that the secondary effect is greater. A positive sign indicates that the secondary effect is in the same direction, and a negative sign indicates that the secondary effect is in the opposite direction.

Applying this general definition to corneal surgery, the CR is the ratio of the change in the power in the meridian 90° away from the surgical meridian (secondary effect) to the power change in the surgical meridian (primary effect).

$$\text{CR} = \frac{\text{Secondary effect}}{\text{Primary effect}}$$

Equation 6: $\text{CR} = \frac{\text{Change } 90^\circ \text{ from surgical meridian}}{\text{Change in surgical meridian}}$

In the three cases under rotation angle, the surgical meridian @ 90° flattened by 1 D (-1.00 @ 90°), while the 180° meridian (90° away) steepened by 1 D ($+1.00$ @ 180°). The CR is

Equation 6: $\text{CR} = \frac{+1.00 \text{ (@ } 180^\circ)}{-1.00 \text{ (@ } 90^\circ)} = -1$

This result simply means that the surgical meridian, and the meridian 90° away, change by equal amounts but in opposite directions. When the CR is -1 , the spherical equivalent of the SIRC must be zero, since the orthogonal meridional changes are

equal and opposite. A CR with magnitude (absolute value) greater than one indicates that more change has occurred 90° away from the surgical meridian; a CR of zero means no change occurred 90° away from the surgical meridian. Positive values indicate that both changes were in the same direction. The theoretical CR for the cornea is -1 .⁹

VII. AVERAGING AXES.

Averaging the axes of cylinders is more complicated than it appears. The primary reason is that when we refer to the axis of a meridian, we usually give only one of the semimeridian values; e.g., when we state the 20° meridian, we really mean the 20° by 200° meridian. The 20° value is actually a semimeridian. Since we are always dealing with the full meridian, we usually omit both semimeridian values because they are evident (always 180° away).

When we begin to take averages of axes, however, the semimeridian that is used can make a difference in the result. For example, suppose two cases had SIRC's at axes of 10° and 170° . Taking the average of these two axes yields 90° [$(10 + 170)/2$]. Clearly, this is not the desired answer; it should be 180° . The reason for the incorrect answer is that the wrong pair of semimeridians was averaged. We should have used 190° ($10^\circ + 180^\circ$) and 170° for the semimeridians to obtain the correct result of 180° (or 0°).

For two meridians, the solution is simple: when semimeridians are averaged, the two values must never be more than 90° apart. When the difference exceeds 90° , one should add 180° to *one* of the semimeridians and then calculate the average. For more than two meridians, the problem is much more complex. In fact, it is not always possible to choose one semimeridian that is less than 90° from *all* the other axes.

We recommend using a different method, in which one calculates the average difference of the axis of the surgically induced cylinder and the surgical meridian. For example, choose the 90° meridian as the surgical meridian for cataract surgery. Express all surgically induced cylinders in the power notation with the same sign for all cylinders (all values must be in either plus-cylinder form or minus-cylinder form). First, calculate the difference of the axis of each cylinder by subtracting the reference axis (e.g., 90°) from the axis of each cylinder (this value is the same as the rotation angle R calculated in Section IV, Rotating Axes). Then calculate the algebraic average of these differences. This average describes the mean difference from the reference meridian. For example, if the result were $+10^\circ$, the average axis of the SIRC would be 100° ($90^\circ + 10^\circ$). If the standard deviation of

this value is small, the result is meaningful. If the standard deviation is large, the result is of little value.

REPORTING AND GRAPHING AGGREGATE RESULTS AND STATISTICS

In general, SIRC's are best visualized when plotted with refractive change on the y-axis and time on the x-axis, as shown in Figure 1. In this graph, we have plotted the spherical equivalent value of the refractive change at each postoperative period. The time axis is shown on a Log scale making the graph a semi-log plot. It is important that the time scale be logarithmic because the decay curves for wound compression, relaxation, and healing are exponential decay functions. In simple terms this means that much more change occurs during the early intervals than later intervals.⁹ When long follow-up periods are plotted on linear time scales, the early periods are so closely compressed that important early changes are difficult to see.

We recommend creating six graphs of the following six parameters determined from the SIRC, as shown in Figures 1 to 9. The six parameters we have chosen are (1) spherical equivalent, (2) magnitude of astigmatism, (3) with-the-wound change (Delta WTW), (4) against-the-wound change (Delta ATW), (5) the coupling ratio, and (6) axis of the SIRC. These six parameters, shown at different postoperative intervals, will accurately describe

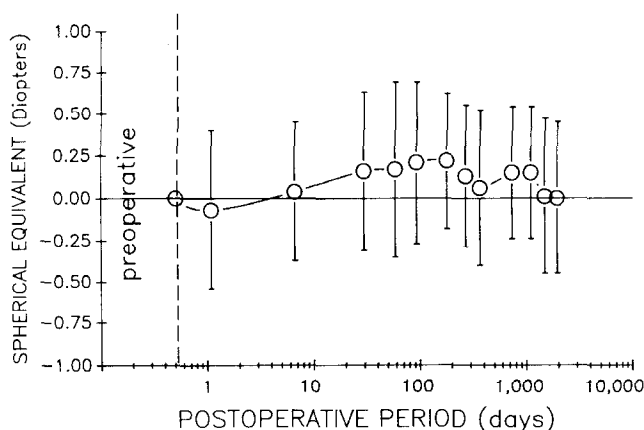


Fig. 1. (Holladay) The spherical equivalent for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). A negative spherical equivalent indicates a decrease or weakening (hyperopic direction) of the corneal power following the procedure, whereas a positive value indicates an increase or strengthening (myopic direction). Notice that the spherical equivalent never changed by more than 0.25 D.

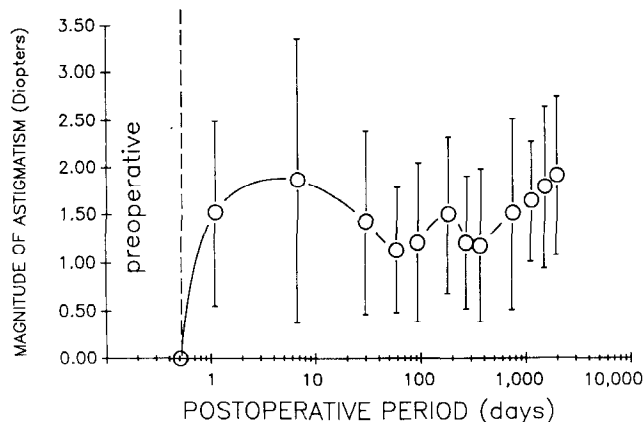


Fig. 2. (Holladay) The magnitude of astigmatism for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The magnitude of the astigmatism for each patient in a given period is averaged. Notice that in this data set the smallest amount of astigmatism was present between 60 and 365 days.

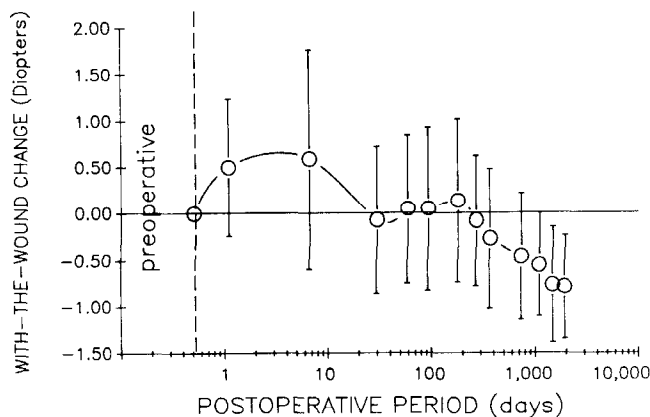


Fig. 3. (Holladay) The with-the-wound change (Delta WTW) component @ 90° for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The graph indicates that the 90° meridian was steepened by approximately 1 D on days 1 and 7, reduced to near zero between 30 and 270 days, and began to flatten progressively after one year.

the SIRC's and will allow comparison with other data sets. These six graphs can be used for one patient or for a set of patients, in which an average value is used for a specific time period. When a group of patients is used, a standard deviation can be calculated for this period and shown as brackets, as depicted in each figure.

As mentioned previously, when examining results and performing further calculations, it is im-

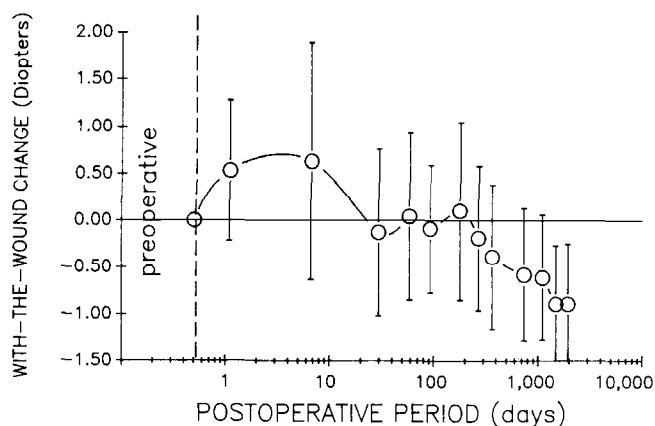


Fig. 4. (Holladay) The with-the-wound change (Delta WTW) peak near 90° for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The graph illustrates the same findings as in Figure 3, but the values for the peaks are 5% to 10% larger than the component values shown in Figure 3.

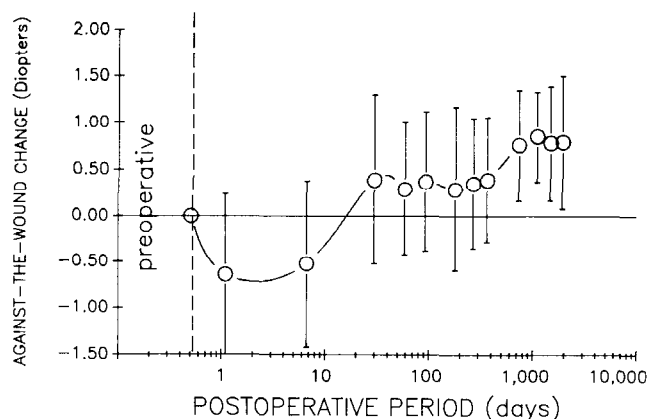


Fig. 5. (Holladay) The against-the-wound change (Delta ATW) component @ 180° for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The graph indicates that the 180° meridian was flattened slightly more than 0.50 D on days 1 and 7, steepened to approximately 0.37 D from day 30 to 365, and continued to steepen to 0.75 D from day 730 to 1,950.

portant to express the SIRC in the *power* notation (e.g., +1 @ 90°).

Figures 1 to 9 have been generated from a data set of 41 patients (TVC) who had extracapsular cataract extraction and IOL implantation with approximately six years follow-up. A 9- to 10-mm limbal incision, centered near 90°, was used. A 10-0 nylon shoelace suture was used to close the wound, and a surgical keratometer was used to adjust the tension of the suture, leaving all patients with approximately 1.50 D @ 90°. No sutures were removed at any time during the postoperative period.

1. Spherical Equivalent (Figure 1)

The spherical equivalent for each case can be determined in the normal manner by taking the sphere plus one-half of the cylinder [S3 + 1/2 (C3)]. The algebraic average of these values yields the overall spherical equivalent for a given period. A negative spherical equivalent indicates a decrease or weakening (hyperopic direction) of the corneal power; a positive value indicates an increase or strengthening (myopic direction) of the corneal power following the procedure. In Figure 1 we see that the spherical equivalent changed less than 0.25 D for the entire postoperative period.

2. Magnitude of Astigmatism (Figure 2)

The magnitude of the astigmatism is determined by taking the absolute value of the cylinder (absolute value of C3). A graph of the magnitude of astigmatism is shown in Figure 2. This graph shows the magnitude of the SIRC over time without re-

spect to axis.

3. With-the-Wound Change (Delta WTW in Figures 3 and 4)

Method 1: Calculating the component @ the surgical meridian (Figure 3). The with-the-wound change component is determined by deciding on the surgical meridian (e.g., cataract surgery @ the 90° meridian) and calculating the component of the SIRC in that meridian. For example, if the change were -1.00 + 2.00 @ 120°, the change in the vertical meridian (@ 90°) would be

Equation 7:
 $\Delta \text{WTW @ } 90^\circ = \text{sphere} + \text{cyl} * [\cos(\text{axis} - 90^\circ)]^2$
 $= -1.00 + 2.00 * [\cos(120^\circ - 90^\circ)]^2$
 $= -1.00 + 2.00 * (0.866)^2$
 $= -1.00 + 2.00 * (0.75)$
 $\Delta \text{WTW @ } 90^\circ = +0.50$

The surgical meridian @ 90° has steepened by 0.50 D. The results of Delta WTW @ 90° following cataract surgery are shown in Figure 3.

Method 2: Use the peak value of the change within 45° of the surgical meridian (Figure 4). Often the SIRC is not exactly at the surgical meridian. The most common reason is that the actual surgical meridian for each case is not exactly at the desired surgical meridian, or the tightest suture in the wound may not be in the center of the incision. When this occurs, the peak change "near" (within 45° of) the surgical meridian will be larger than the component @ 90°. Although the difference is usu-

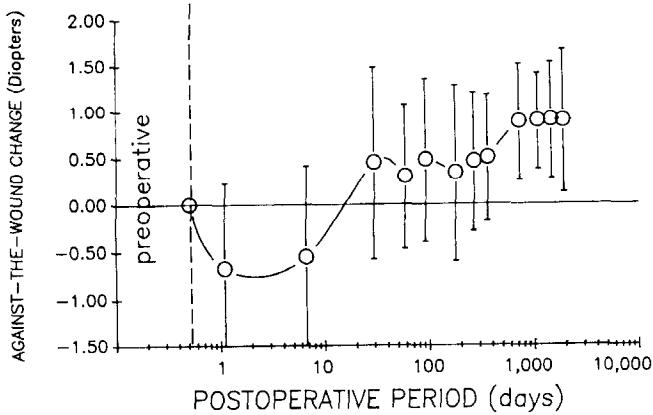


Fig. 6. (Holladay) The against-the-wound change (Delta ATW) peak near 180° for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The graph illustrates the same findings as in Figure 5, but the values for the peaks are 5% to 10% larger than the component values shown in Figure 5.

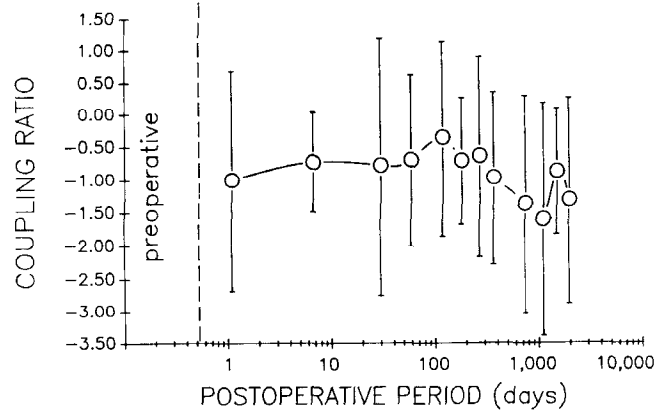


Fig. 8. (Holladay) The CR using the peaks near 180° and near 90° for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The CR using the peak values is almost identical to the results in Figure 7 using the component values.

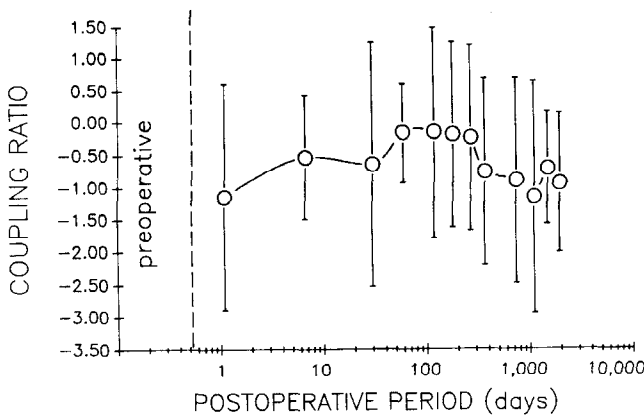


Fig. 7. (Holladay) The CR using the components @ 180° and @ 90° for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The CR for each patient is calculated dividing the component @ 180° by the component @ 90°. The average of these values is plotted for each period. The CR oscillates about -1.00, indicating that the change in corneal power @ 90° has been accompanied by a power change at 180° of equal magnitude but opposite sign.

ally small in large data sets, using the peak value near the surgical meridian yields the maximum change near the surgical meridian, which may be more meaningful for some procedures. The "peak" value is determined from the crossed cylinder form of SIRC using the cylinder nearest the surgical meridian. Writing the SIRC of $-1.00 + 2.00 @$

120° in the crossed cylinder form, we have $-1.00 @ 30°$ and $+1.00 @ 120°$. The peak WTW is therefore $+1.00$, since this is the cylinder nearest 90°.

4. Against-the-Wound Change (Delta ATW in Figures 5 and 6)

Method 1: Calculating the component 90° away from the surgical meridian (Figure 5). The against-the-wound change component is determined by the change 90° away from the surgical meridian (e.g., if cataract surgery was @ the 90° meridian, the Delta ATW would be at 180°). We must now calculate the component of the SIRC in that meridian (@ 180°). For example, if the change were $-1.00 + 2.00 @ 120°$, the change in the horizontal meridian (@ 180°) would be

$$\text{Equation 8:} \\ \Delta \text{ATW @ } 180^\circ = \text{sphere} + \text{cyl} * [\cos(\text{axis} - 180^\circ)]^2$$

$$= -1.00 + 2.00 * [\cos(120^\circ - 180^\circ)]^2 \\ = -1.00 + 2.00 * (0.500)^2 \\ = -1.00 + 2.00 * (0.250)$$

$$\Delta \text{ATW @ } 180^\circ = -0.50$$

The horizontal meridian has flattened by 0.50 D. The results of Delta ATW @ 90° following cataract surgery are shown in Figure 4.

Method 2: Use the peak value of the change more than 45° away from the surgical meridian (Figure 6). The peak value of the against-the-wound change may not be exactly 90° from the surgical meridian for the same reasons as the with-the-

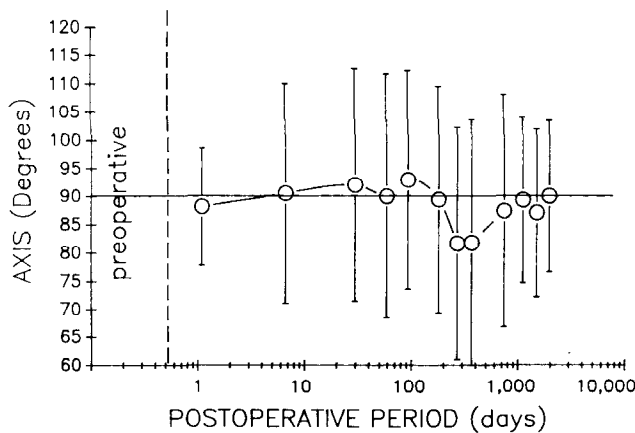


Fig. 9. (Holladay) The axis of the SIRC for the TVC 41-patient data set is shown for various postoperative periods (O is the mean and I is the standard deviation). The average axis was always within 10° of the 90° surgical meridian. Although it is not statistically significant, at 270 and 365 days the axis was nearer 80° than 90°.

wound changes were not at 90°. The peak values are slightly larger than the value of the component. The “peak” value is determined from the crossed cylinder form of SIRC using the cylinder farthest away from the surgical meridian. Writing the SIRC of $-1.00 + 2.00 @ 120^\circ$ in the crossed cylinder form, we have $-1.00 @ 30^\circ$ and $+1.00 @ 120^\circ$. The peak ATW is therefore -1.00 , since this is the cylinder nearest 180° .

5. Coupling Ratio (Figures 7 and 8)

The CR is calculated as previously described in section VI. When a surgical meridian has been chosen, the CR will be defined by:

$$CR = \frac{\text{Delta ATW}}{\text{Delta WTW}}$$

The CR may be calculated using the values for ATW and WTW from method 1 (Figure 7) or method 2 (Figure 8). Theoretically, the results should be identical, but in our experience using the peak values (method 2) yields slightly more consistent results because the peak values are larger, reducing the effect of round-off error in the calculations.

For the example above,

Equation 9:

$$CR = \frac{\text{Delta ATW} - \text{component @ } 180^\circ}{\text{Delta WTW} - \text{component @ } 90^\circ}$$

$$CR = \frac{-0.50 (@ 180^\circ)}{+0.50 (@ 90^\circ)}$$

$$CR = -1.00$$

This result indicates that the change 90° away from the surgical meridian was equal and opposite to the surgical meridian. The vertical surgical meridian has been steepened by $0.50 D$, while the horizontal meridian has flattened by $0.50 D$. Occasionally, values for Delta WTW may become very small (less than $0.25 D$), indicating that there has been very little change in the surgical meridian. Since small values near zero are close to the limit of the accuracy of refraction and keratometry, using such small values sometimes leads to erroneously large CRs. When determining averages, we recommend not using the values for the CR when Delta WTW is less than $0.25 D$.

Calculating the CR using the peak values “near” the surgical meridian and “away” from the surgical meridian are performed in the same way:

Equation 10:

$$CR = \frac{\text{Delta ATW} - \text{peak near } 180^\circ}{\text{Delta WTW} - \text{peak near } 90^\circ}$$

For the previous example,

$$CR = \frac{-1.00 (@ 120^\circ)}{+1.00 (@ 30^\circ)}$$

$$CR = -1$$

Notice that in this example the results of the CR using “peak” values and the component values are equal.

6. Axis of the SIRC (Figure 9)

The average axis of the SIRC on the 41 patient data set (TVC) is shown in Figure 9. The method used to calculate the values at each time period is described under Averaging Axes in section VII.

CONCLUSION

We hope that our techniques of calculating and reporting the SIRC will be implemented in software systems that will be available to those who are not computer enthusiasts. Figures 1 to 9 show the important changes that are taking place in the cornea as a function of time. These types of analyses should provide us with the answers to the optimal wound size for cataract surgery and will help us refine many of our keratorefractive procedures.

REFERENCES

1. Stokes GG. 19th Meeting of the British Association for the Advancement of Science, 1849. *Trans Sect* 1850; 10
 2. Naylor EJ. Astigmatic difference in refractive errors. *Br J Ophthalmol* 1968; 52:422-425
 3. Jaffe NS, Clayman HM. The pathophysiology of corneal astigmatism after cataract extraction. *Trans Am Acad Ophthalmol Otolaryngol* 1975; 79:615-630
 4. Cravy TV. Calculation of the change in corneal astigmatism following cataract extraction. *Ophthalmic Surg* 1979; 10(1):38-49
 5. Michaels DD. *Visual Optics and Refraction; a Clinical Approach*. St Louis, CV Mosby, 1980; 62
 6. Holladay JT, Gruber AJ, Lewis JW. Refining toric soft contact lens prescriptions. *CLAO J* 1984; 10:326-331
 7. Holladay JT, Rubin ML. Avoiding refractive problems in cataract surgery. *Surv Ophthalmol* 1988; 32:357-360
 8. Rubin ML. *Optics for Clinicians*, 2nd ed. Gainesville, Fla, Triad Scientific Pub, 1974; 105
 9. Waring GO III, ed. *Refractive Keratotomy for Myopia and Astigmatism*. St Louis, Mosby-Year Book, 1992; 1101-1105, 1139
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