

Evaluating and Reporting  
Astigmatism Outcomes  
for  
Individual and Aggregate Data

by

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7/17/2006

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1

Spheroequivalent (SEQ)

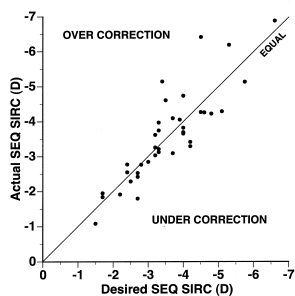
$$SEQ = Sphere + \frac{1}{2} * Cylinder$$

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6

Equivalency Plot

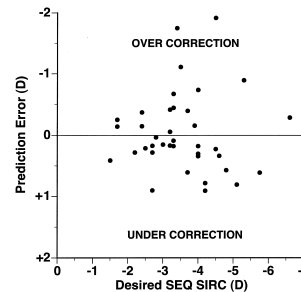


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7

Prediction Error (D) Plot



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8

DEFOCUS EQUIVALENT

“DEQ” = |SEQ| + 1/2 |cyl|

-0.50 + 1.00 x 90 = +0.50

-5.00 + 10.00 x 90 = +5.00

Proportional to Blur Circle/ $\sqrt{a}$

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9

Spectacle and Corneal  
Astigmatism NOT EQUAL

MYOPIA -- Corneal Astig Lower

HYPEROPIA -- Corneal Astig Higher

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10

### Vertexing Refraction from Spectacle to Cornea

$$REF_C = \frac{1000 * REF_S}{1000 - REF_S * Vertex(mm)}$$

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11

### Three RX Forms @ Spectacle Plane

- ① Plus cyl: -5.34 + 1.44 X 65°
- ② Minus cyl: -3.90 - 1.44 X 155°
- ③ Cross cyl: -3.90 X 65° and -5.34 X 155°

Vertex: 14 mm

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12

### Vertexing Cross Cylinder Form from Spectacle to Cornea

$$REF_{C1} = \frac{1000 * (-3.90)}{1000 - (-3.90) * 14} = -3.70D$$

$$REF_{C2} = \frac{1000 * (-5.34)}{1000 - (-5.34) * 14} = -4.97D$$

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13

### Three RX Forms @ Corneal Plane

- ① Plus cyl: -4.97 + 1.27 X 65°
- ② Minus cyl: -3.70 - 1.27 X 155°
- ③ Cross cyl: -3.70 X 65° and -4.97 X 155°

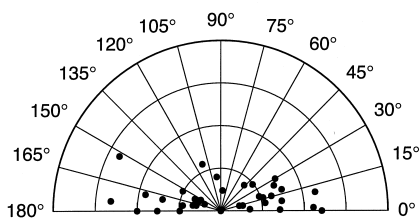
Vertex: 0 mm

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14

### Single Angle Minus Cylinder Power Plot

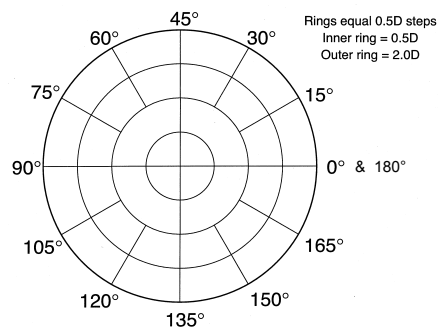


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15

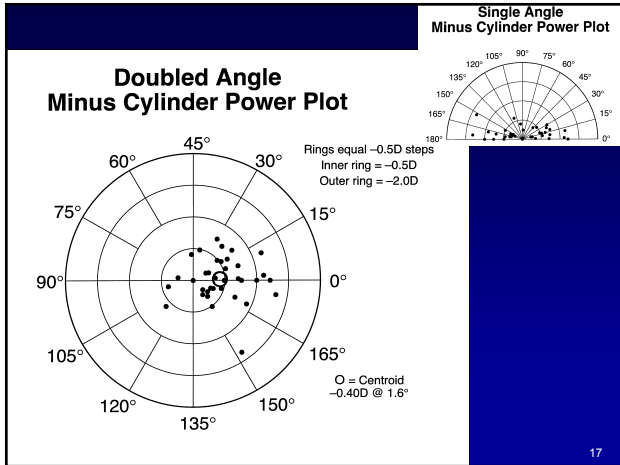
### Doubled Angle Plot



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16



### Computing Mean Astigmatism

Mean of X = 
$$\frac{\sum_{i=1}^n x_i}{n}$$

Mean of Y = 
$$\frac{\sum_{i=1}^n y_i}{n}$$

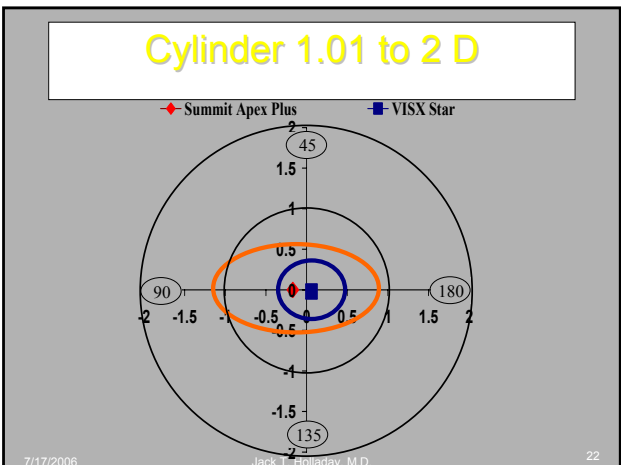
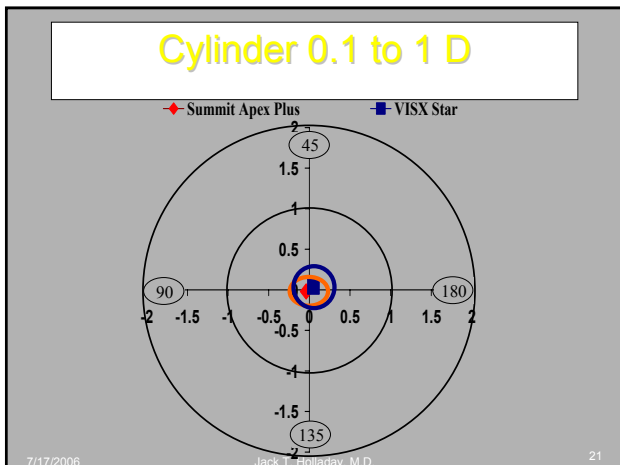
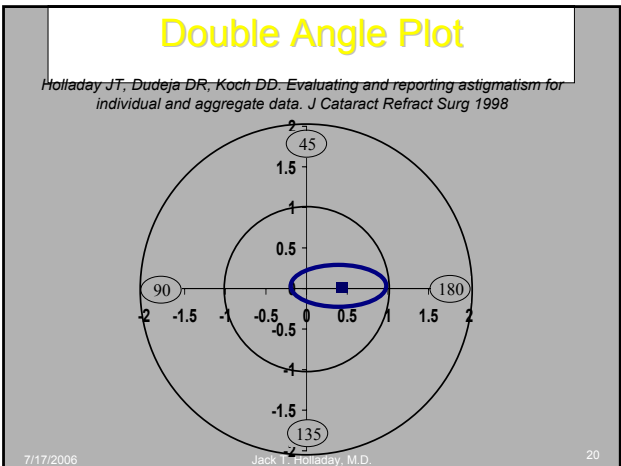
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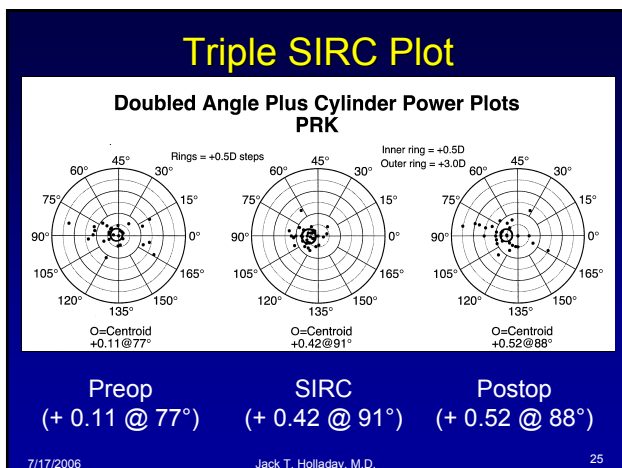
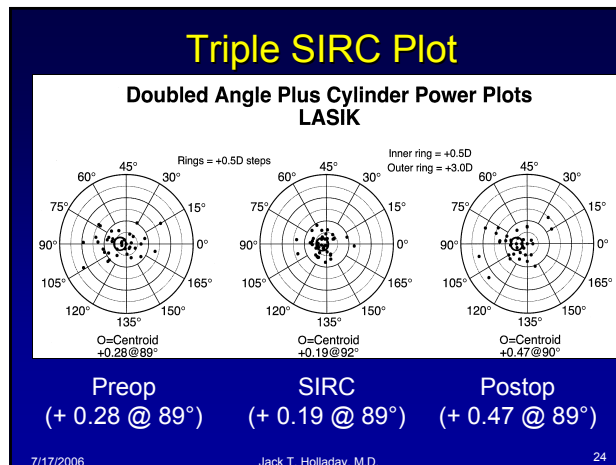
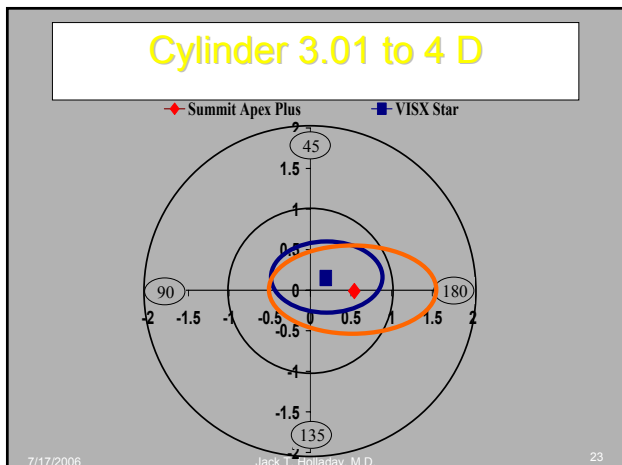
### Computing Standard Deviation Astigmatism

S.D. of X = 
$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

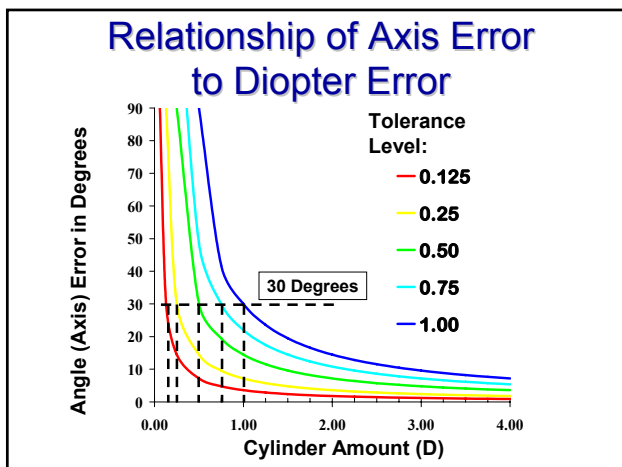
S.D. of Y = 
$$\sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}}$$

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- ### Conclusion
- 1 Astigmatism Analysis must use doubled angles & DEQ
  - 2 Cylinders and axes must be converted to Cartesian Coordinates (X and Y)
  - 3 SIRC and Prediction Error are the appropriate parameters to evaluate astigmatic results
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### Dioptic Error vs. Angular Error for a 1.00 D of astigmatism

Angle Error (°)	Dioptic Error (D)	% Error
0°	0.00	0%
15°	0.52	52%
30°	1.00	100%
45°	1.41	141%
60°	1.73	173%
75°	1.93	193%
90°	2.00	200%

Dioptic Error = 2 \* Cyl \* sin (angular error)

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### Converting from Polar (Cyl, Axis) to Cartesian (X, Y) Coordinates

$$X = \text{Cylinder} * \text{Cos}(2 * \text{axis})$$

$$Y = \text{Cylinder} * \text{Sin}(2 * \text{axis})$$

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32

### Converting from Cartesian (X, Y) Coordinates to Polar (Cyl, Axis)

$$\text{Cylinder} = \sqrt{X^2 + Y^2}$$

$$\text{Angle} = \frac{1}{2} * \text{Arc tan} \left( \frac{Y}{X} \right)$$

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33

### Final Axis of Mean Astigmatism

- IF  $X \& Y > 0$   
THEN AXIS = Angle
- IF  $X < 0$   
THEN AXIS = Angle +  $90^\circ$
- IF  $X > 0 \& Y < 0$   
THEN AXIS = Angle +  $180^\circ$

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34

### Example Conversion from Cartesian to Polar

$$X = + 0.399 \quad \& \quad Y = + 0.022$$

$$\text{Cylinder} = \sqrt{.399^2 + .022^2} = 0.40D$$

$$\text{Angle} = \frac{1}{2} * \text{Arc tan} \left( \frac{0.022}{0.399} \right) = 1.6^\circ$$

$$\text{Since } X \& Y > 0 \quad \underline{+0.40 \times 1.6^\circ}$$

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35

! Thank you for your Attention !



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38