power indicates the steepening or flattening and the torsional power the rotation over the surgical meridian.[2]

Astigmatism is a mathematical concept, and adherence to mathematical formalism will allow for subsequent statistical analysis using well-known methods.[2] The net astigmatism in the form of astigmatic direction and magnitude is used to characterize a single astigmatism in daily clinical practice. However, net astigmatisms cannot be used for calculations because they do not represent a vector format.[2] Therefore, the description of centroids in Figures 1 and 2 as net astigmatisms with standard deviations (SDs) is not correct.[1] Because net astigmatisms are not vectors, SDs cannot be calculated from and should not be associated with this format. The centroid is the combined mean of the x component and y component, while the spread is described with their variances.

Figure 1 in the guest editorial graphically shows the precision of 4 different corneal topographers.[1] Why was a statistical evaluation not shown? Two variances can be compared with an F test and multiple variances with a homogeneity test.

Regarding astigmatism analysis for cornea–based refractive surgery, the *Journal of Refractive Surgery* (*JRS*) standard for reporting astigmatism outcomes of corneal refractive surgery included nonvector methods and the use of single-angle plots.[3] Despite robust criticism,[4] this standard was also adopted by the *Journal of Cataract & Refractive Surgery*® (*JCRS*). In a later joint editorial between *JRS* and *JCRS*, some of the same principles were also endorsed for IOL-based surgery.[5] Thus, at present *JCRS* advocates 3 antagonistic sets of methods and terminologies for analysis of SIA.[1,3,5] This is an untenable position for a scientific journal.

A comprehensive analysis of SIA should be performed with similar methods and terminologies for corneal-based and IOL-based surgical procedures. Previous standards for reporting astigmatism outcomes have failed because they were narrowly based on the ideas of single researchers. To elaborate enduring common standards, all relevant experts should be invited to participate. Would it be possible for JCRS to provide the editorial framework for such consensus seeking?

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Reply: We have great respect and admiration for all of the excellent work Dr. Næser has contributed, especially in the area of the mathematics for astigmatism analysis. The intent of the fourth editorial was to show the double-angle plot, centroid, and 95% confidence ellipses for any type of astigmatic vectors (preoperative/postoperative astigmatism, astigmatism prediction error, and SIA) and to provide a tool in Excel (Microsoft Corp.) that will provide the statistical values and figures.[A] We have followed the details in his thesis[1] to the letter as we will explain in detail below.

All angles are doubled according to Stokes,[2–4] and for a cylinder of magnitude M and axis Φ, the 2 vector components are:

$$x = C_0 = M\cos 2\varnothing \tag{1}$$

$$y = C_{45} = M\sin 2\varnothing \tag{2}$$

We used x and y for the components, rather than the C_0 and C_{45}, because most Journal readers and authors are not mathematicians and we were trying to keep everything as simple as possible. The terms meridional and torsional power for the x and y components in the double-angled plots also might be confusing. (We do agree that, with properly chosen reference planes, the meridional power indicates the steepening or flattening, and the torsional power indicates the rotation over the surgical meridian.)

The centroid is the arithmetic mean of x and y:

$$\text{Mean of } C_0 = \frac{\sum C_0}{n} = \textit{Mean of } x = \frac{\sum x}{n} = \bar{x} \tag{3}$$

$$\text{Mean of } C_{45} = \frac{\sum C_{45}}{n} = \textit{Mean of } y = \frac{\sum y}{n} = \bar{y} \tag{4}$$

In mathematics and physics, the centroid or geometric center of a plane figure is the arithmetic mean position of all the points in the figure. The centroid is not intended to be synonymous with the net astigmatism. When the centroid is not at the origin, it represents a constant offset of the aggregate data. If an adjustment is made (as with the improvement of a toric calculator) so that the centroid is moved to the origin without changing the total variance of the data, there will be an improvement on the outcomes. In the double-angle plot figures, the x and y values for the centroid have been converted back to their polar equivalent, which is shown at the bottom of the plot. The resulting vector is the magnitude of the average astigmatism and meridian or axis of the average astigmatism.

$$M_{\text{Mean}} = \sqrt{\left(\frac{\sum C_0}{n}\right)^2 + \left(\frac{\sum C_{45}}{n}\right)^2} = \sqrt{\left(\frac{\sum x}{n}\right)^2 + \left(\frac{\sum y}{n}\right)^2} = \sqrt{\bar{x}^2 + \bar{y}^2} \tag{5}$$

$$\varnothing = \text{Arctan}\frac{M_{\text{Mean}} - \frac{\sum C_0}{n}}{\frac{\sum C_{45}}{n}}$$

$$= \text{Arctan}\frac{M_{\text{Mean}} - \frac{\sum x}{n}}{\frac{\sum y}{n}} = \text{Arctan}\frac{M_{\text{Mean}} - \bar{x}}{\bar{y}} \quad (6)$$

The value shown as $\pm$ x.xx after the centroid in the figures is the total SD of x and y, which is the square root of the total variance of x and y. This is done because the total SD is in the same units as the original data (diopters), rather than in diopters squared.

$$\text{Total Variance} = \text{Variance of } x + \text{Variance of } y \quad (7)$$

$$\text{Total Variance} = \frac{\sum (x - \bar{x})^2}{n - 1} + \frac{\sum (y - \bar{y})^2}{n - 1} \quad (8)$$

$$\text{Total Standard Deviation} = \sqrt{\text{Total Variance}}$$

$$= \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1} + \frac{(y - \bar{y})^2}{n - 1}} \quad (9)$$

The total variance and SD are independent of the rotation and translation of the coordinate system for the confidence ellipses, so they can be calculated from the original vector components. We used the exact methods and formulas for bivariate analysis described for determining the tolerance or normal ellipse to describe the 95% confidence ellipse for the centroid and the 95% confidence ellipse of the dataset, which are analogous to the use of standard errors of the mean and SDs, respectively, for univariate data.[5] We realize that 2 variances might be statistically compared with an F test and multiple variances with a homogeneity test. Per Dr. Næser's recommendation, we will add this to the double-angle plot tool so that investigators can generate P values comparing the variances of various datasets.

The mean absolute value of the magnitude of the astigmatism (M) is indeed a scalar value, and it is noted at the bottom of Figure 3, *A*, in our editorial.

$$\text{Mean absolute astigmatism} = \frac{\sum M}{n} \quad (10)$$

It is particularly helpful to surgeons in evaluating their postoperative residual astigmatism because it is completely independent of the axis.

The authors below are in complete agreement that, to establish enduring common standards, all relevant experts should be invited to participate, and we hope that JCRS will provide the editorial framework for such a consensus-seeking endeavor.—*Jack T. Holladay, MD, Douglas D. Koch, MD, Adi Abulafia, MD, Li Wang, MD, PhD, Warren Hill, MD*

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