Proper Method for Calculating Average Visual Acuity

Jack T. Holladay, MD, FACS

Calculating the average visual acuity and standard deviation on a series of patients is not difficult, but has been done incorrectly in most studies.[1] The basic problem relates to the difference between the arithmetic and geometric mean for a set of numbers. For the correct average visual acuity, the geometric mean must be used, which gives significantly different values than the arithmetic mean.

Modern visual acuity charts are designed so that the letter sizes on each line follow a geometric progression (ie, change in a uniform step on a logarithmic scale).[2-4] The accepted step size has been chosen to be 0.1 log unit steps, which is equivalent to letter sizes changing by a factor of 1.2589 between lines. This standard gave rise to the LogMAR (log of the minimum angle of resolution) notation, as shown in Table 1.

A geometric progression of lines on the visual acuity chart was chosen because it parallels the way our visual system functions. If patient #1 has a visual acuity of 20/20 and patient #2 has a visual acuity of 20/40, we conclude that patient #1 has two times better visual acuity than patient #2 because he or she can recognize a letter twice as small. Once we have chosen to compare vision as a ratio using a reference visual angle (20/20), a geometric progression results and a geometric mean must be calculated for a meaningful result.

Notice in Table 1 that the only values that increase linearly are the line numbers and the LogMar notation. The Snellen acuity, decimal acuity, and visual angle all increase by the geometric factor of 1.2589. Once we decide that equal steps in visual acuity measurement are geometric and not arithmetic, we must use the appropriate geometric mean to compute the correct average (Figure).

In Table 1 and the Figure, we see that line 0 is the 20/20 Snellen acuity that corresponds to the LogMAR value zero, since 20/20 is the standard. We also see that line 10 is the 20/200 Snellen visual acuity that corresponds to a LogMAR value of +1.00 (ten times or 1 log unit worse than 20/20). Intuitively, it would appear that halfway between line 0 and line 10 would be line 5, or 20/63. This is the correct average, because geometrically it is halfway between 20/200 and 20/20.

The two incorrect methods would be to take the arithmetic average of the Snellen denominators or the arithmetic average of the decimal acuity or visual angle. In the example above, if we average the Snellen denominators, we obtain 20/110 {(200+20)/2}, which would underestimate the true average acuity by more than two lines. If we average the decimal acuities, we obtain 0.55 {(0.1 + 1.0)/2}, which is between 20/40 and 20/32 and overestimates the average visual acuity by more than two lines.

The simplest method for computing the proper average visual acuity from any notation is to convert the value to the LogMAR equivalent and then take the average of the LogMAR values. The easiest way to compute the LogMAR value is to convert to decimal notation and then take the negative of the logarithm, eg, 20/20 = 1 and the log of 1 is 0, and 20/200 = 0.10 and the negative of the log is +1.0. The average of 0 and +1.0 is 0.5 LogMAR units. Converting back from the logMAR value of 0.5, the corresponding visual acuity is 20/63, the correct geometric average.

From the University of Texas Medical School at Houston, Texas.
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Correspondence: Jack T. Holladay, MD, FACS, Houston Eye Associates Building, 2855 Gramercy, Houston, Texas 77025.

Table 1
Corresponding Visual Acuities

Line No.	Snellen Equivalent (feet)	(meters)	Decimal Equivalent (minutes)	Visual Angle	LogMAR* Equivalent
-3	20/10	6/3	2.00	0.50	-0.30
-2	20/12.5	6/3.75	1.60	0.63	-0.20
-1	20/16	6/4.8	1.25	0.80	-0.10
0	20/20	6/6	1.00	1.00	0.00
1	20/25	6/7.5	0.80	1.25	+0.10
2	20/32	6/6.4	0.63	1.60	+0.20
3	20/40	6/12	0.50	2.00	+0.30
4	20/50	6/15	0.40	2.50	+0.40
5	20/63	6/18.9	0.32	3.15	+0.50
6	20/80	6/24	0.25	4.00	+0.60
7	20/100	6/30	0.20	5.00	+0.70
8	20/125	6/37.5	0.16	6.25	+0.80
9	20/160	6/48	0.13	8.00	+0.90
10	20/200	6/60	0.10	10.00	+1.00
11	20/250	6/75	0.08	12.50	+1.10
12	20/320	6/96	0.06	16.00	+1.20
13	20/400	6/120	0.05	20.00	+1.30
.	.	.	.	.	.
.	.	.	.	.	.
20	20/2000†	6/600	0.01	100.00	+2.00
30	20/20000§	6/6000	0.001	1000.00	+3.00

** Log of Minimum Angle of Resolution*
† 20/2000 = count fingers at 2 feet
§ 20/20000 = hand motion at 2 feet

The formulas for going from decimal to Logmar and back are:

$$\text{LogMAR} = - \text{Log (Decimal Acuity)} \quad (1)$$
$$\text{Decimal acuity} = \text{antilog} (- \text{LogMAR}) = 10^{-\text{LogMAR}} \quad (2)$$

Two other considerations occur when evaluating sets of visual acuity measurements: 1) what to do with values of count fingers, hand motion, light perception, etc., and 2) how to compute the correct value if the patient did not read all of the letters on the line completely.

Count Fingers, Hand Motion, Light Perception, No Light Perception

Count fingers at a given distance can be converted to a Snellen equivalent by assuming that the fingers are approximately the size of the elements of a 200 letter. Therefore, a person who can count fingers at 20 feet would have approximately 20/200 vision.[5] A person able to count fingers at 2 feet would have 2/200 vision or the equivalent of 20/2000. This value is somewhat conservative because the hand against a white coat is much lower contrast than a black letter on a white background. Also, the examiner usually uses four or less fingers, making the number of forced choices less than the number of Snellen optotypes.[10]

From studies performed in our low vision clinic, hand motion at a given distance is ten times worse than count fingers, ie, a person who can detect hand motion at 20 feet has approximately 20/2000 Snellen visual acuity equivalent. A person who has hand motion at 2 feet would have an equivalent Snellen acuity of 20/20,000.

Light perception with and without projection and no light perception are not actually visual acuity measurements, but simply the detection of a stimulus. These cases should be excluded and described in the materials and methods section of a manuscript.

Patient Cannot Read Entire Line

It is common for visual acuity sets to include values in which the patient did not read all of the letters on a single line correctly. Although recording the last line that was read completely or the majority of letters (three out of five) is an acceptable method, it reduces the precision of the measurement—similar to rounding off laboratory measurements. A more accurate method is to interpolate between the values of the LogMar acuity using the

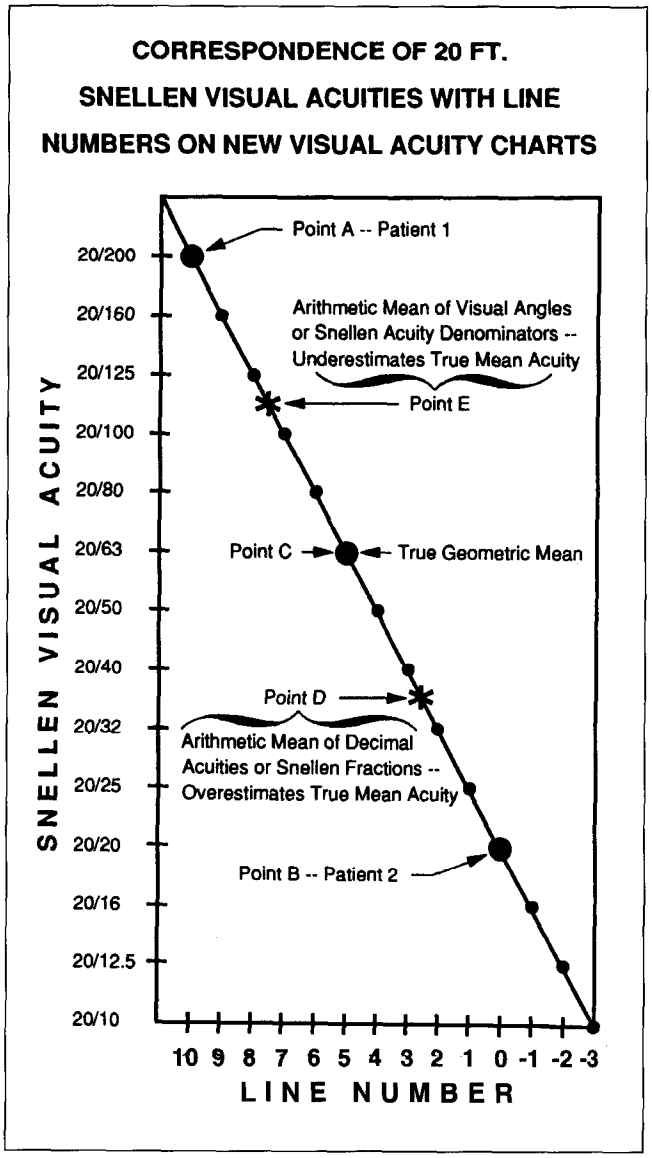

Figure: The true geometric mean visual acuity between Patient 1 with visual acuity of 20/200 (Point A) and Patient 2 with visual acuity of 20/20 (Point B) is 20/63 (Point C). The incorrect arithmetic mean obtained by taking the average of the decimal visual acuities or Snellen fractions is 20/36 (Point D). The arithmetic mean obtained by taking the average of the Snellen visual acuity denominators or visual angles is 20/110 (Point E). Arithmetic means severely overestimate or underestimate the true geometric mean visual acuity. (From Holladay JT, Prager TC. *Am J Ophthalmol* 1991;111:372-373)

fraction of the number of letters read correctly on a visual acuity line.

For example, suppose our acuity chart had five letters on each visual acuity line and the patient read all of the letters on the 20/50 (LogMar +0.4) line, but only three of the five letters on the 20/40 (LogMar +0.3) line. Three-fifths (3/5 = 0.6) of the way from LogMar +0.4 to +0.3 is LogMar +0.34. The LogMar value of +0.34 is the correct value for this patient's visual acuity. For studies that involve large databases, where converting these values manually is tedious, we have published the formulas that allow direct conversion from the Snellen acuity value to the interpolated LogMar value.[6] These formulas only work if there are an equal number of letters on a line, as there are on the Bailey-Lovie[4] visual acuity chart and other standardized charts.[2,3] Unfortunately, if the number of letters on the acuity chart are not equal on each line (as occurs on many projected and wall charts), then a table must be created that shows the conversion interpolation for each line, and a single formula is not possible.

Table 2
Visual Acuity Data Set for Seven Theoretical Eyes

Eye No.	Measured Visual Acuity*	Snellen Equivalent	Decimal Equivalent	LogMAR Equivalent
1	20/10	20/10	2.0	-0.30
2	20/10-2	20/10-2	2.0-2	-0.26
3	20/40	20/40	0.5	+0.30
4	20/40+3	20/40+3	0.5+3	+0.24
5	20/200	20/200	0.1	+1.00
6	CF† at 2 ft	20/2000	0.01	+2.00
7	HM§ at 2 ft	20/20000	0.001	+3.00
Mean		20/142	0.141	+0.85
Standard deviation		± 11.5 lines	± 11.5 lines	±0.115

** Bailey-Lovie visual acuity chart with five letters on each line*
† Count fingers
§ Hand motion

Example Calculations

Once the LogMar value for the visual acuity of each patient has been obtained, then statistical analyses of the data set can be performed. All statistical calculations (mean, standard deviation, standard error of the mean, correlation coefficient, etc.) must be calculated using LogMar values for visual acuity. Performing these analyses using any other value for visual acuity will lead to erroneous results.[7]

Table 2 presents a data set to illustrate the correct calculations and serves as a check for an investigator to use to validate his or her calculation method. The average value and standard deviation are calculated using LogMar values. The average LogMar acuity was 0.85 and the standard deviation was 1.15, normally expressed as 0.85 ± 1.15.

To determine the equivalent decimal acuity for the average we must use equation (2):

Decimal visual acuity = $10^{-LogMar}$ = $10^{-0.85}$ = 0.141
Snellen visual acuity denominator =
20/decimal acuity = 20/0.141 = 142
Snellen visual acuity = 20/142

The only meaningful conversion of the standard deviation in LogMar units is to use lines of visual acuity. Since each line of a standardized visual acuity chart increases by 0.1 log units, a standard deviation of ±1.15 log units is equivalent to ±11.5 lines (1.15/0.1). In this data set of seven patients, the mean visual acuity and standard deviation are 0.85 ± 1.15 LogMar units, 0.141 ± 11.5 lines in decimal units and 20/142 ± 11.5 lines in Snellen units.

Other statistical calculations such as correlation coefficients, Student's *t*-test, analysis of variance (ANOVA), etc. should be performed using the LogMar values for the mean and standard deviation. Using these techniques will provide meaningful analyses of data sets and allow valid comparisons of different data sets.

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